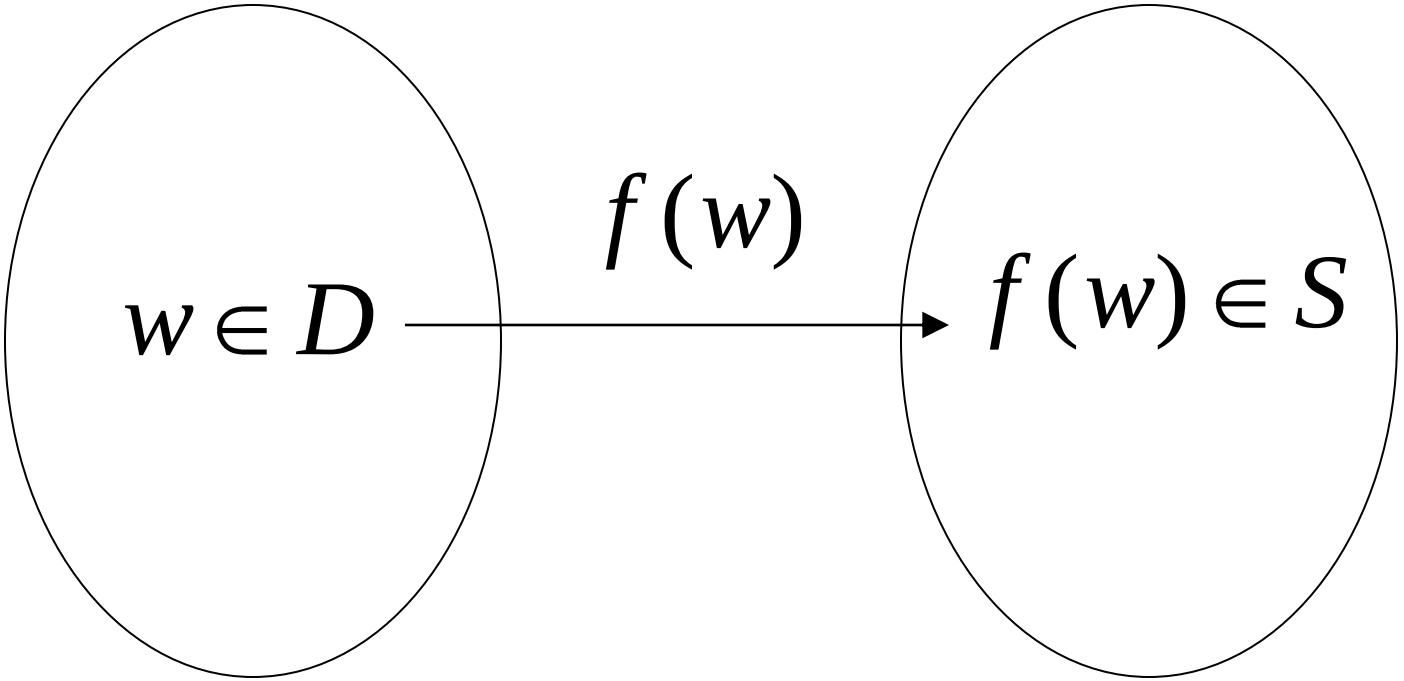
Calcolare funzioni con Macchine di Turing

Una funzione $f(w)$ ha:

Dominio: D

Range: S



Una funzione puo' dipendere da piu' variabili:

Esempio: La funzione addizione

$$f(x, y) = x + y$$

Dominio: insieme degli interi

Notazione decimale: 5

Notazione binaria: 101

Notazione unaria: 11111

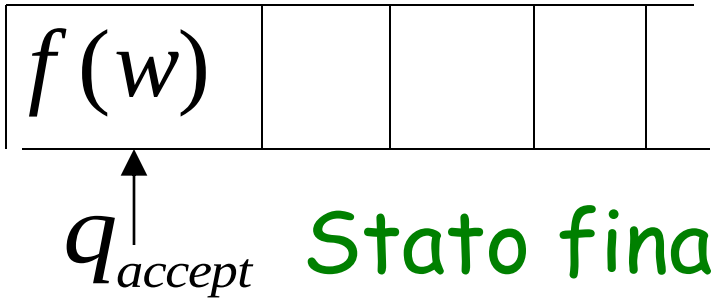
La rappresentazione **unaria** e' piu' semplice

Idea:

Una funzione f e' calcolabile se esiste una macchina di Turing M tale che:

Configurazione iniziale

Configurazione finale



Per ogni $w \in D$ D = Dominio

Esempio

La funzione $f(x, y) = x + y$ e' calcolabile

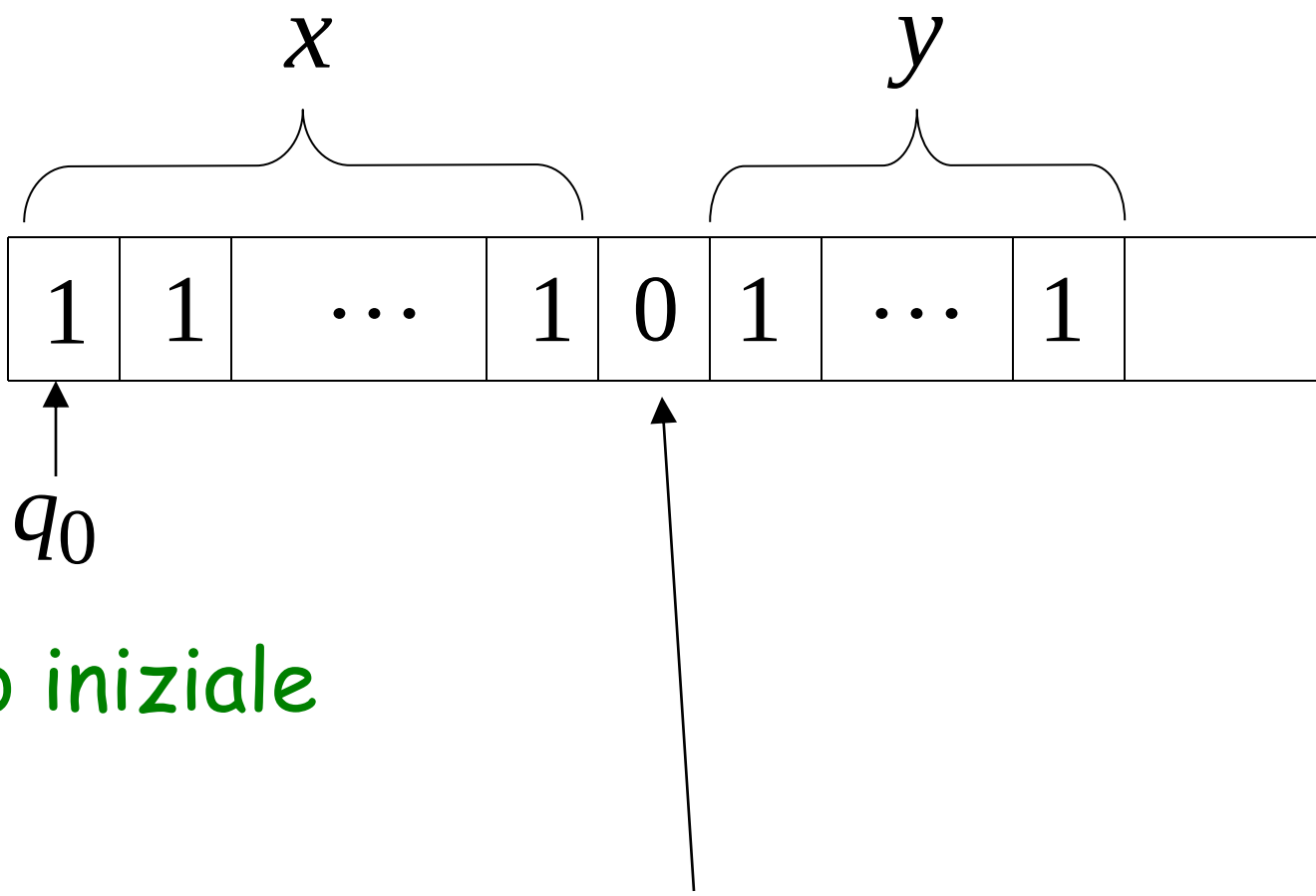
x, y sono interi

Macchina di Turing:

stringa input: $x0y$ unario

stringa output: $xy0$ unario

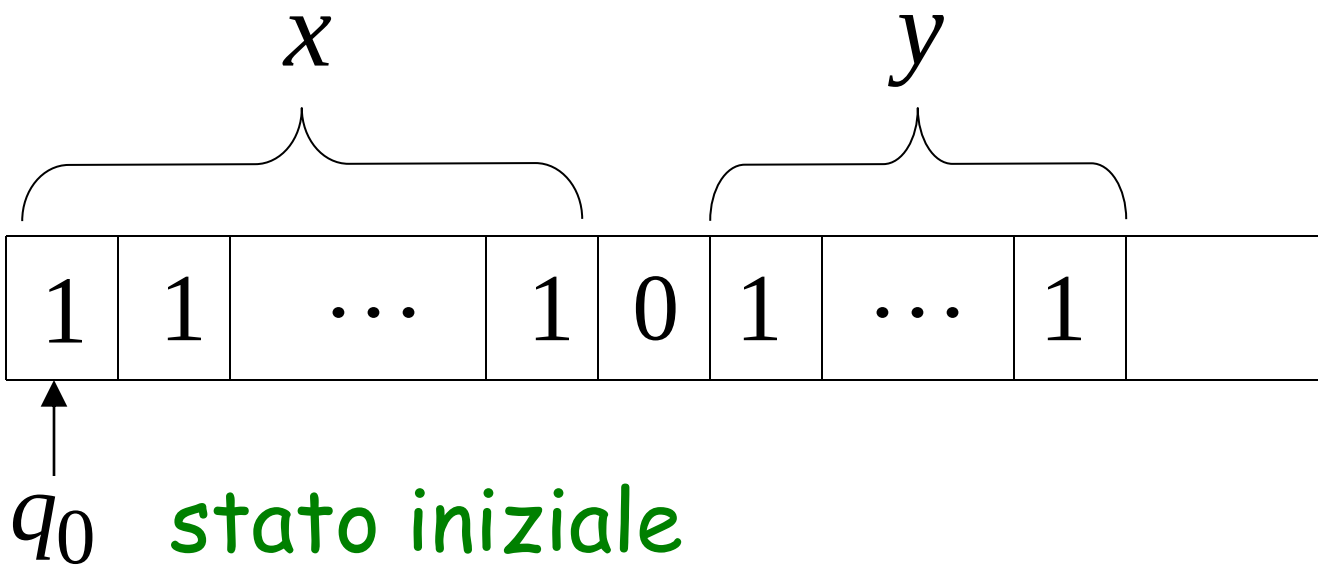
Inizio



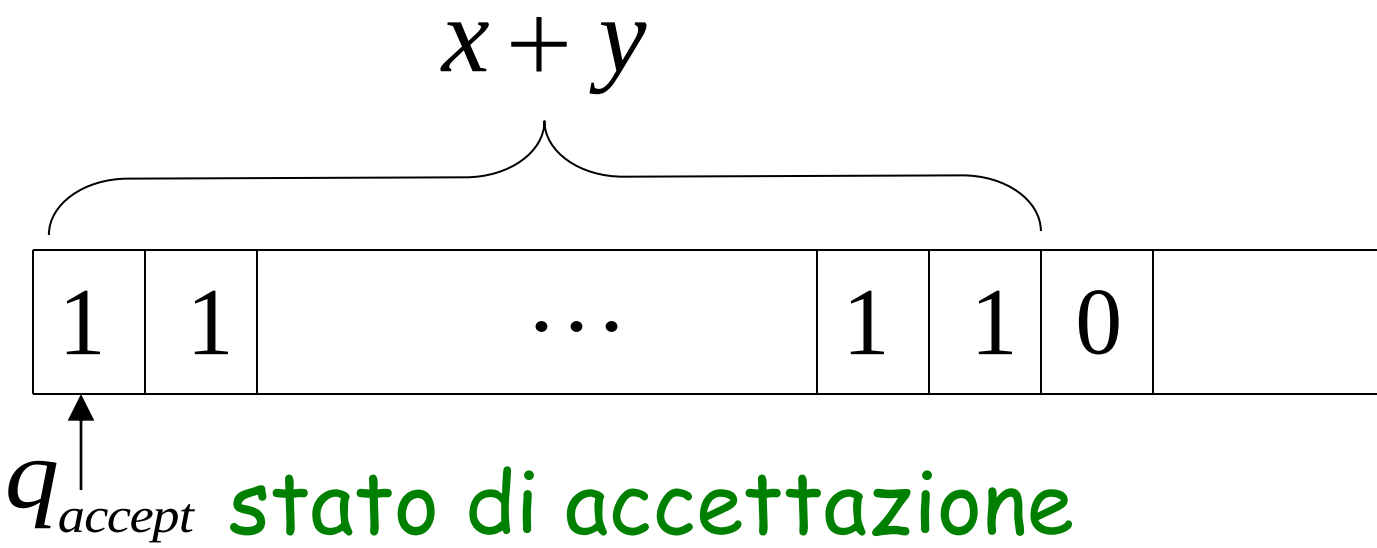
Stato iniziale

Lo 0 e' il carattere che separa i due numberi

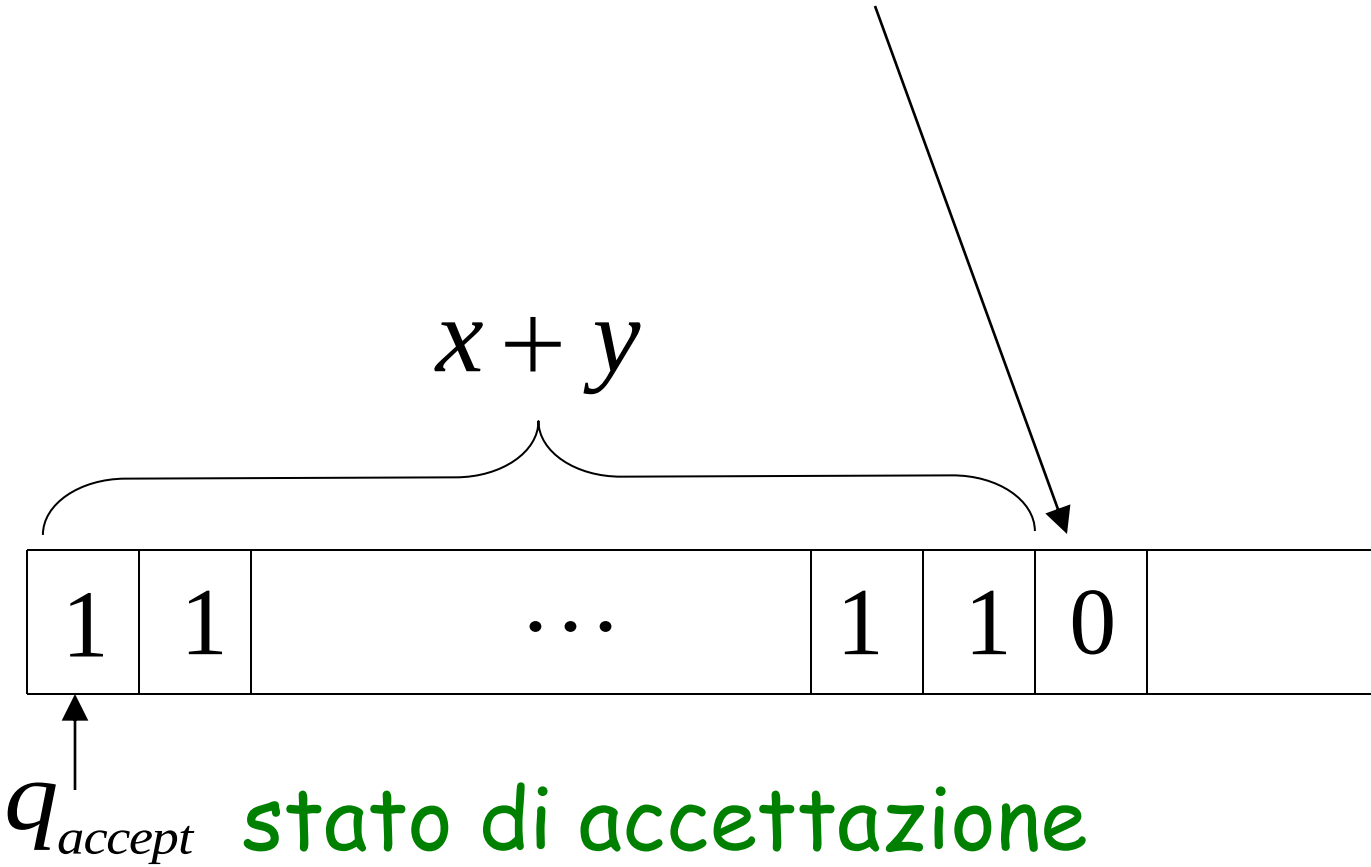
Inizio


x
y
1
1
⋯
1
0
1
⋯
1
q_0
stato iniziale

Fine

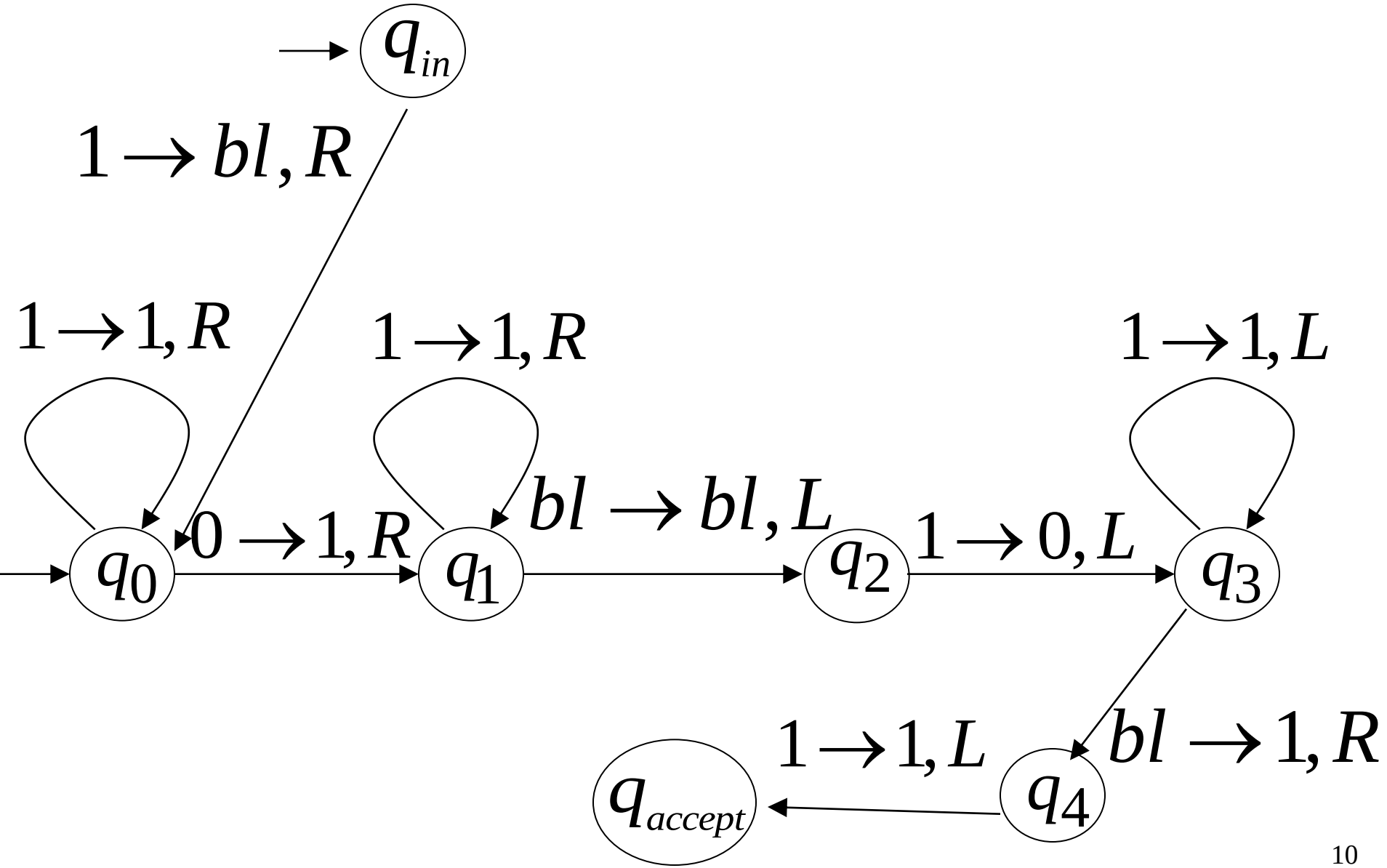

x + y
1
1
⋯
1
1
0
q_{accept}
stato di accettazione

lo 0 aiuta se volessimo usare
il risultato per altre operazioni



Fine

Macchina di Turing per $f(x,y) = x + y$

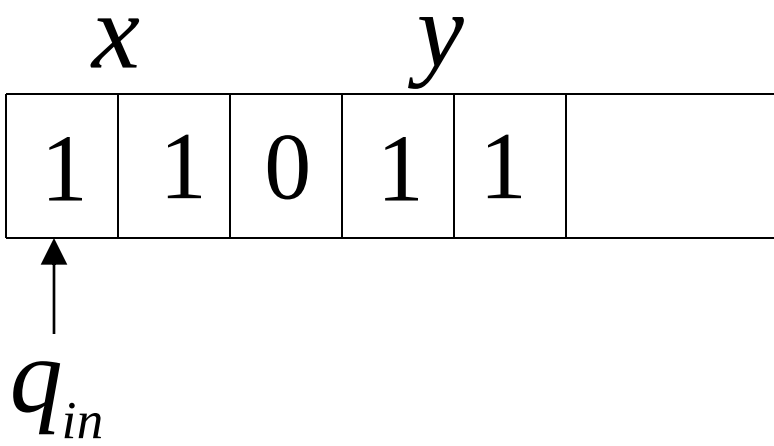


Esempio di esecuzione:

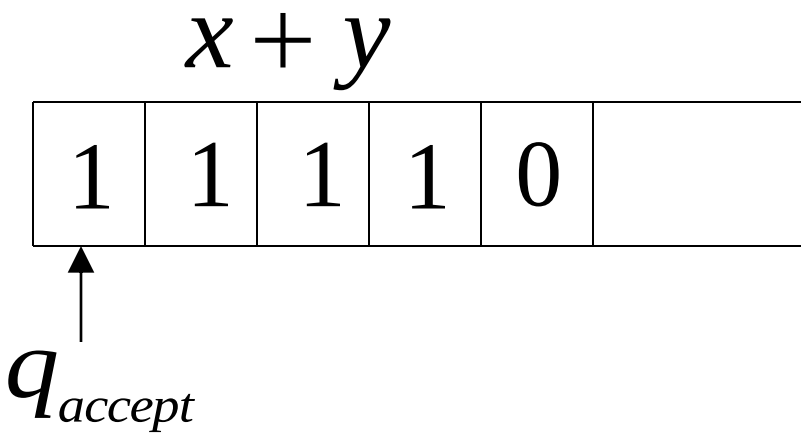
$x = 11$ (2)

$y = 11$ (2)

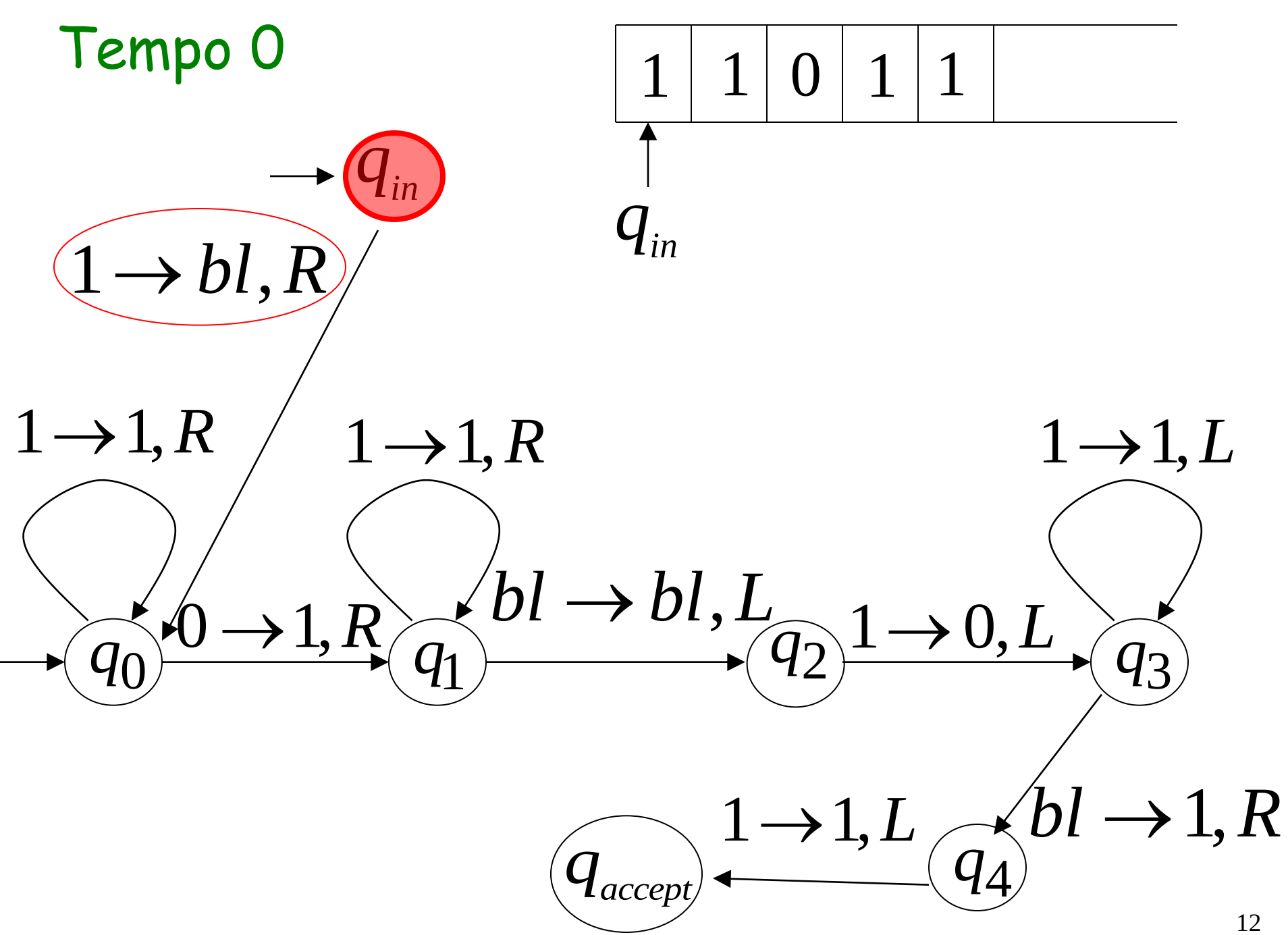
Tempo 0



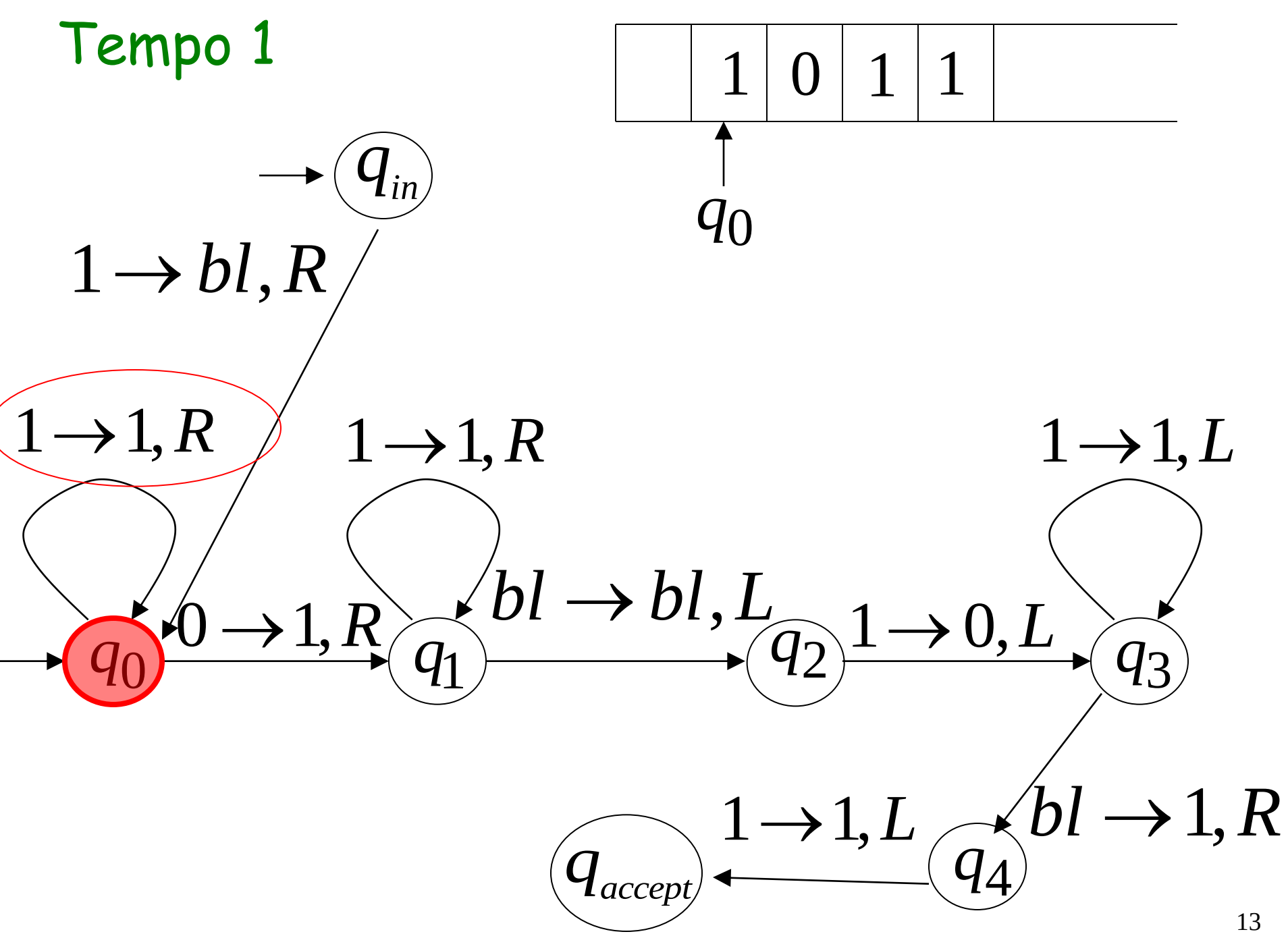
Risultato Finale



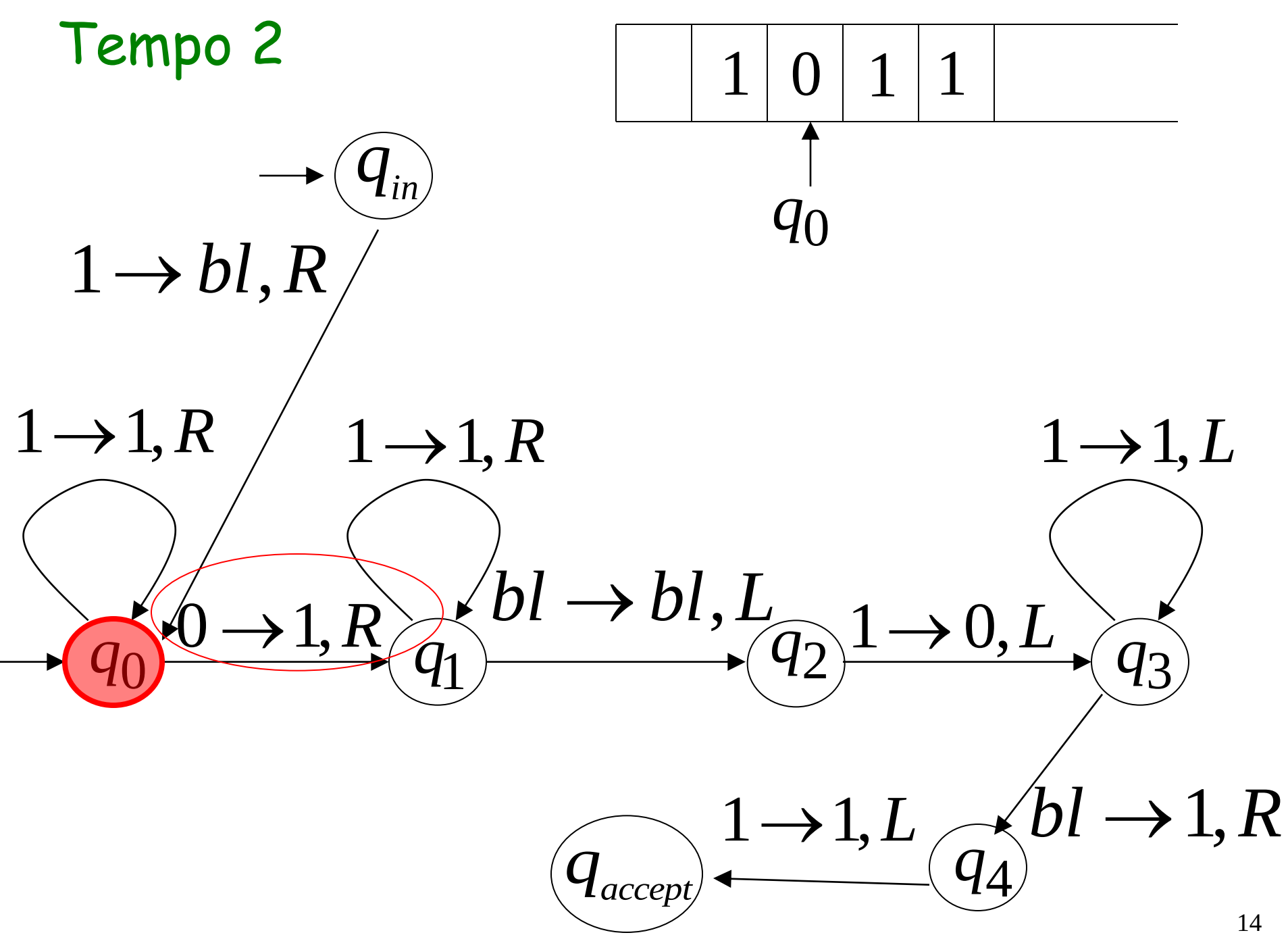
Tempo 0


1 1 0 1 1
q_{in}
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_4
q_{accept}
$1 \to 1, L$
$bl \to 1, R$

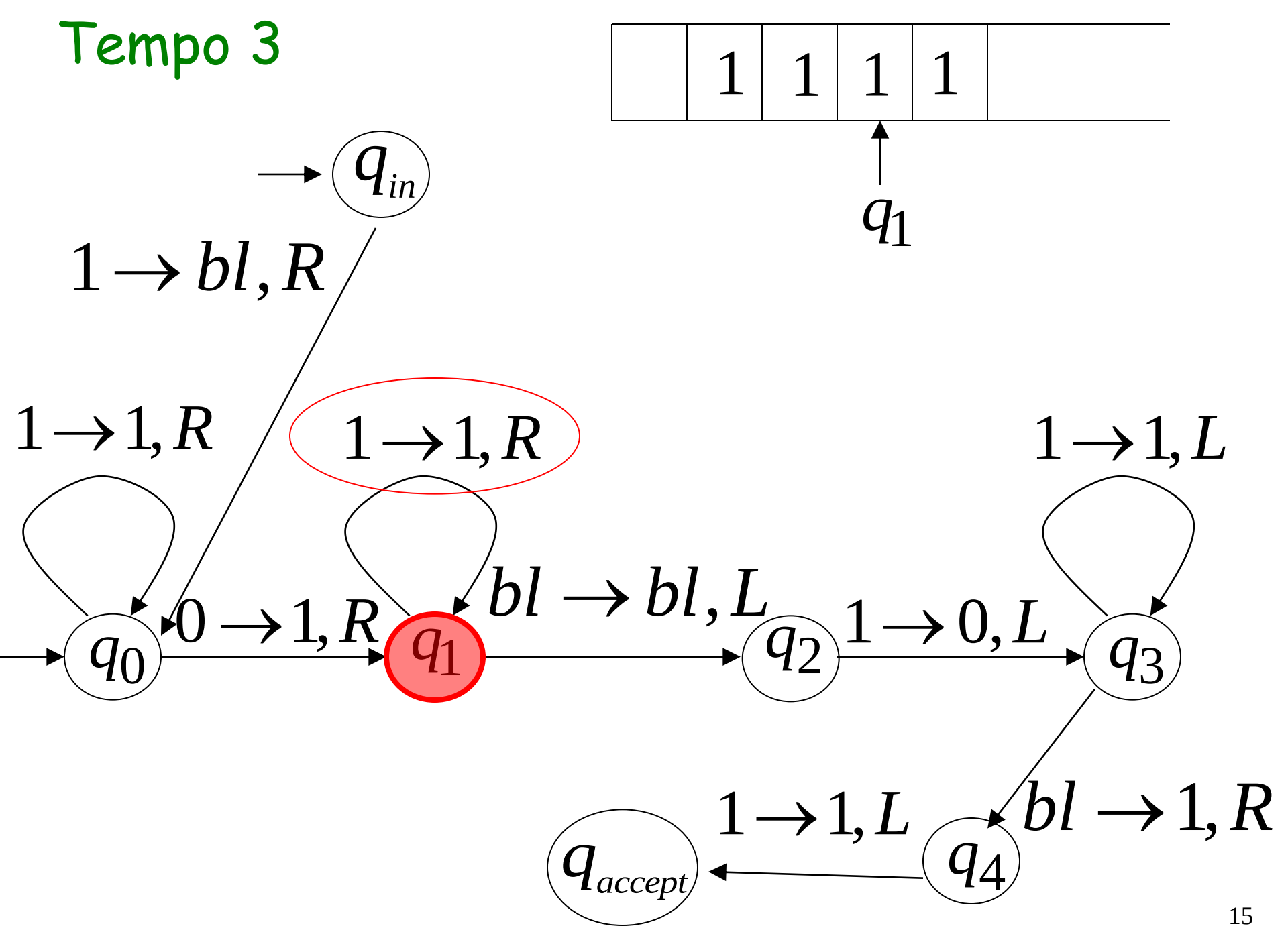
Tempo 1


1 0 1 1
q_0
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_4
q_{accept}
$1 \to 1, L$
$bl \to 1, R$

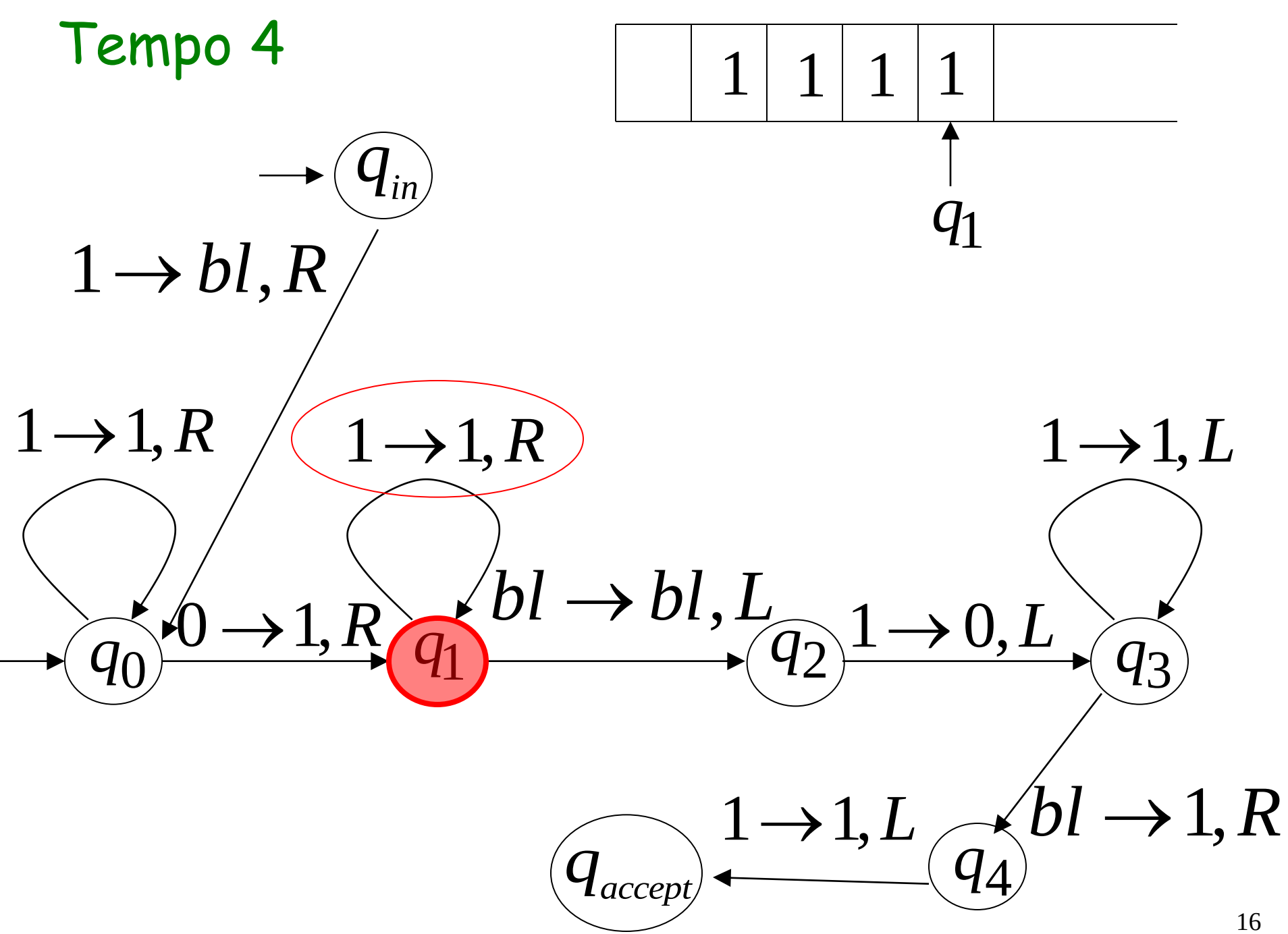
Tempo 2


1 0 1 1
q_0
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_4
q_{accept}
$1 \to 1, L$
$bl \to 1, R$

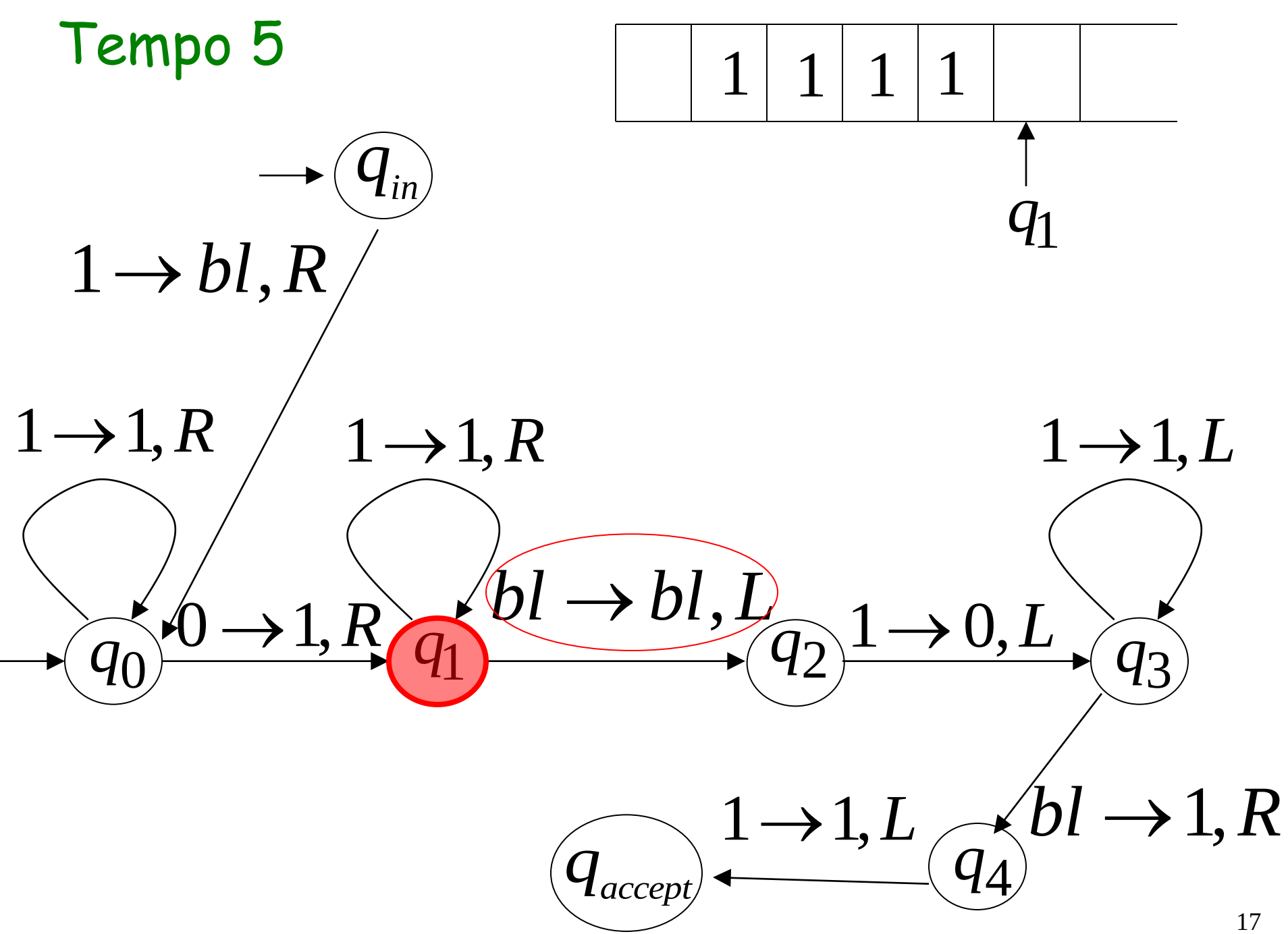
Tempo 3


1 1 1 1
q_1
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$bl \to bl, L$
$0 \to 1, R$
$1 \to 0, L$
q_0
q_1
q_2
q_3
$bl \to 1, R$
$1 \to 1, L$
q_{accept}
q_4

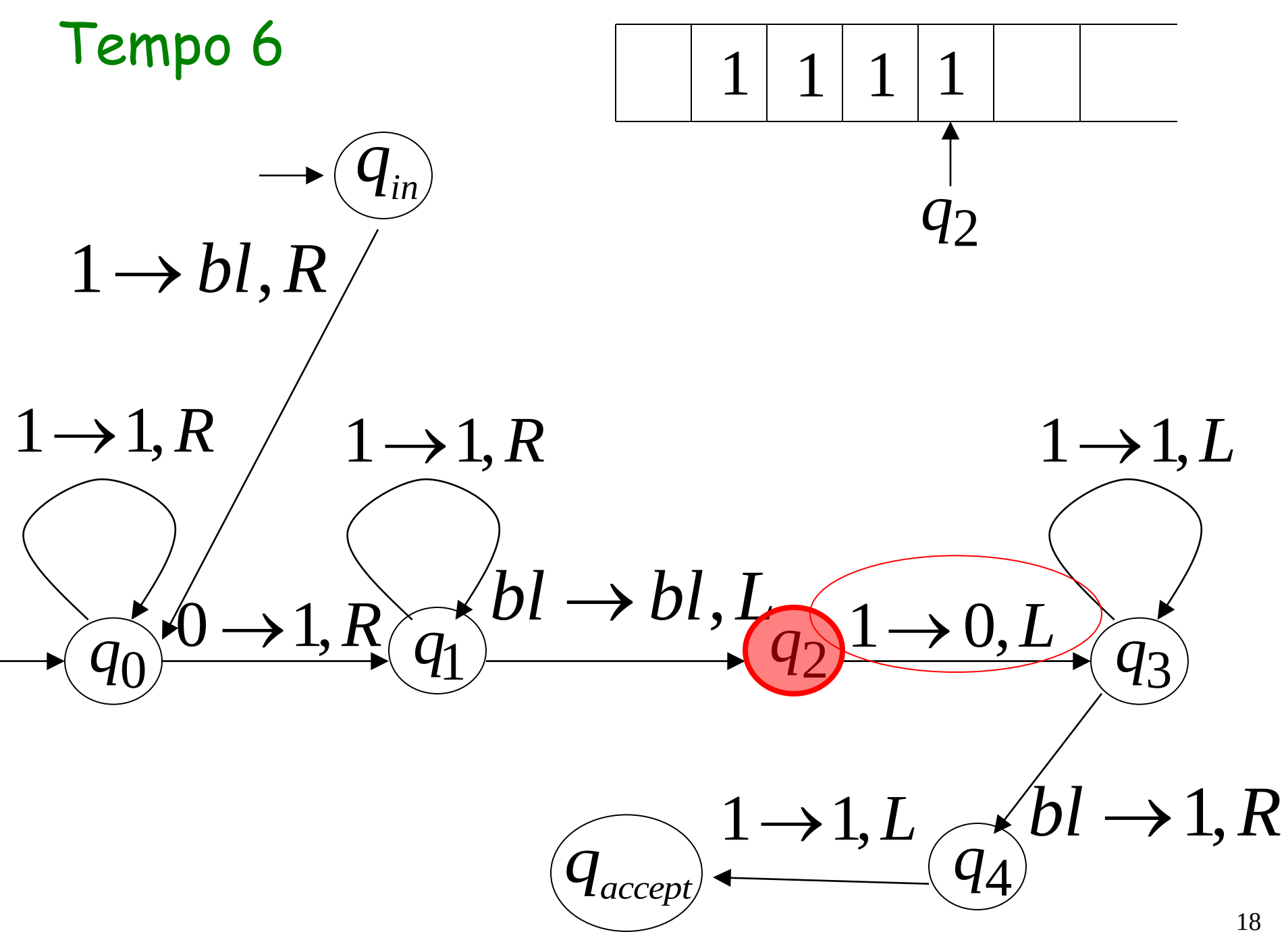
Tempo 4


1 1 1 1
q_1
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$bl \to bl, L$
$0 \to 1, R$
$1 \to 0, L$
q_0
q_1
q_2
q_3
$bl \to 1, R$
$1 \to 1, L$
q_{accept}
q_4

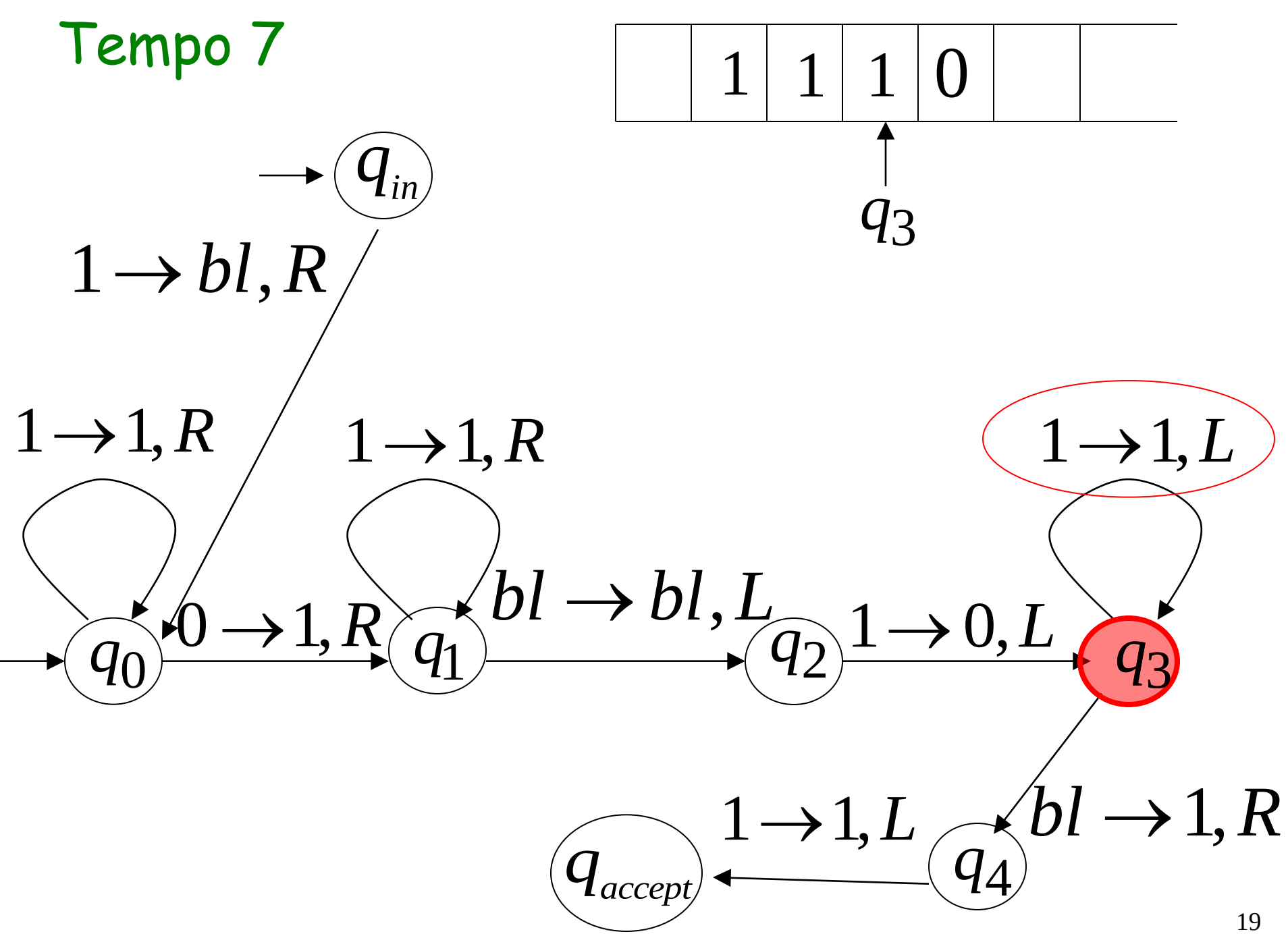
Tempo 5


1 1 1 1
q_1
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$bl \to bl, L$
$0 \to 1, R$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_4
q_{accept}
$1 \to 1, L$
$bl \to 1, R$

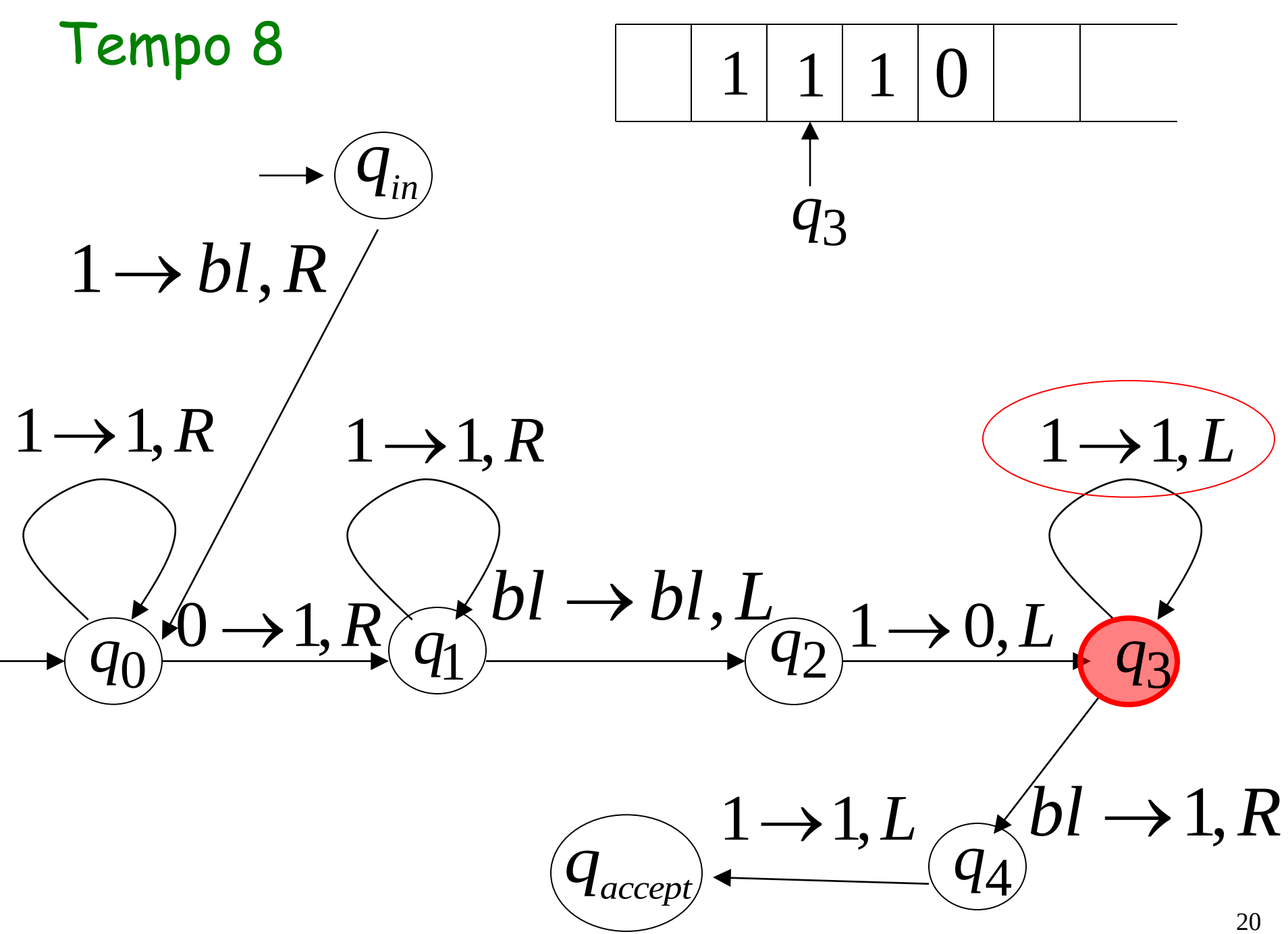
Tempo 6


1 1 1 1
q_2
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_4
q_{accept}
$1 \to 1, L$
$bl \to 1, R$

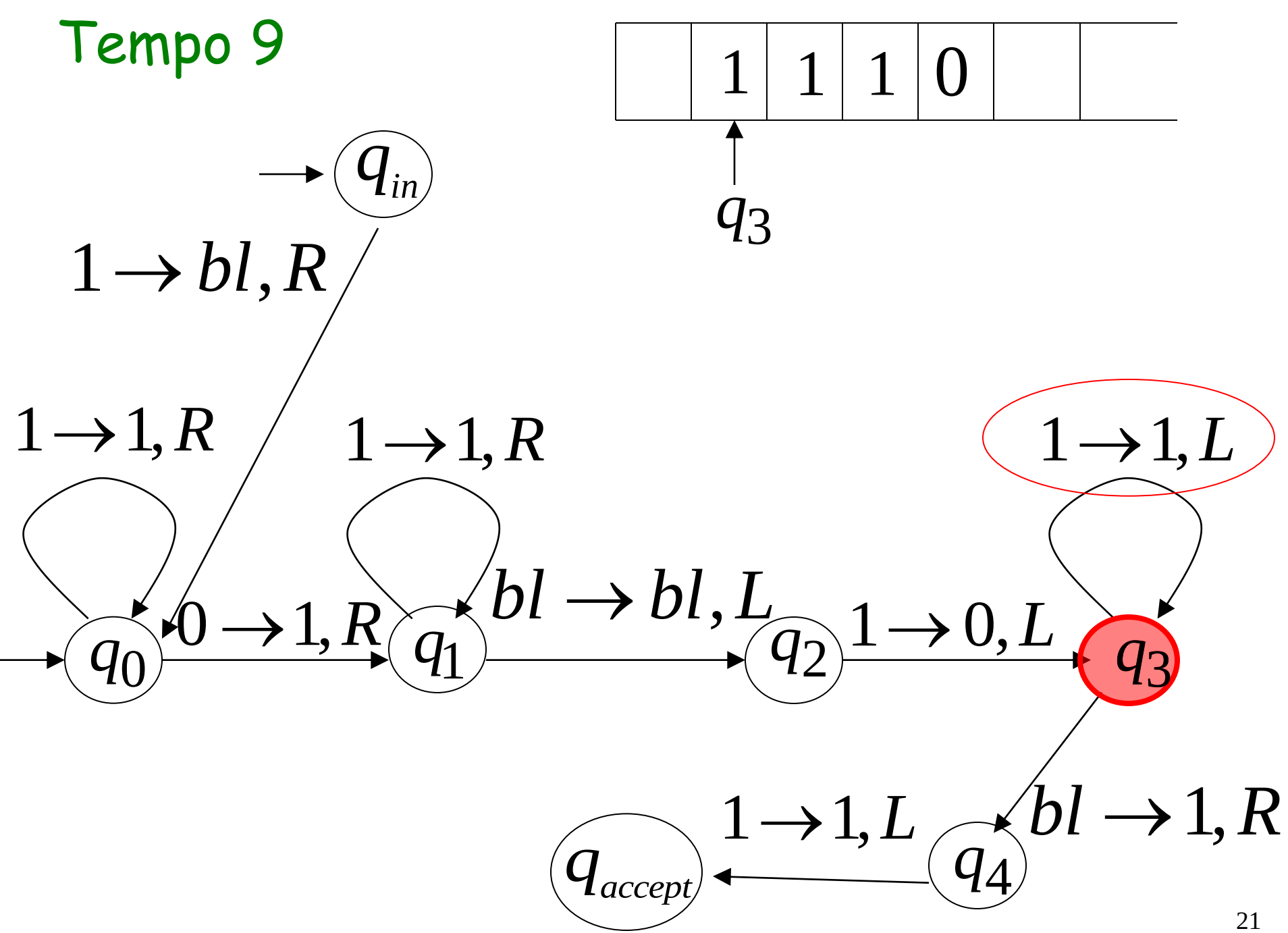
Tempo 7


1 1 1 0
q_3
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_{accept}
q_4
$1 \to 1, L$
$bl \to 1, R$

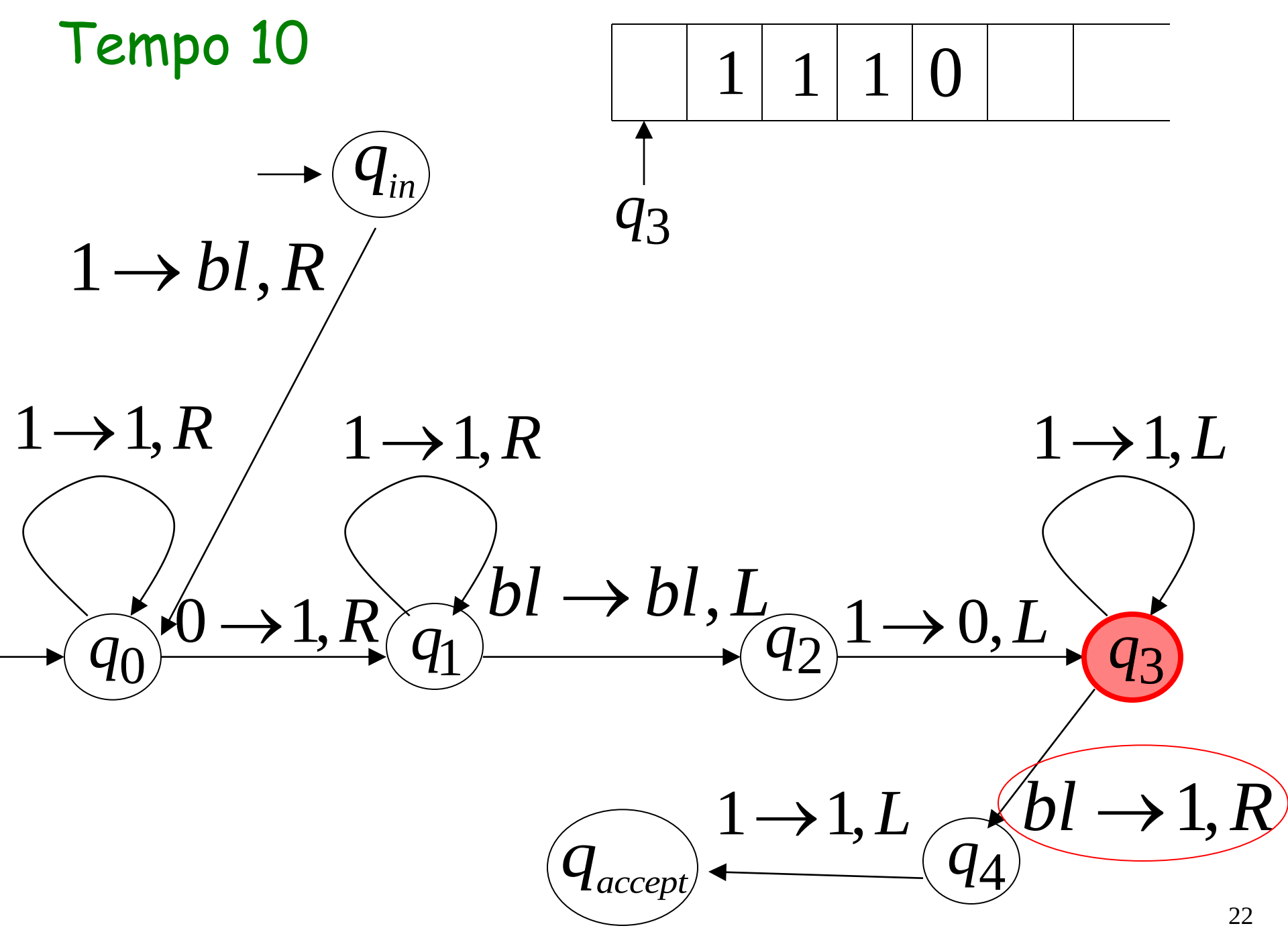
Tempo 8


1 1 1 0
q_3
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_{accept}
q_4
$1 \to 1, L$
$bl \to 1, R$

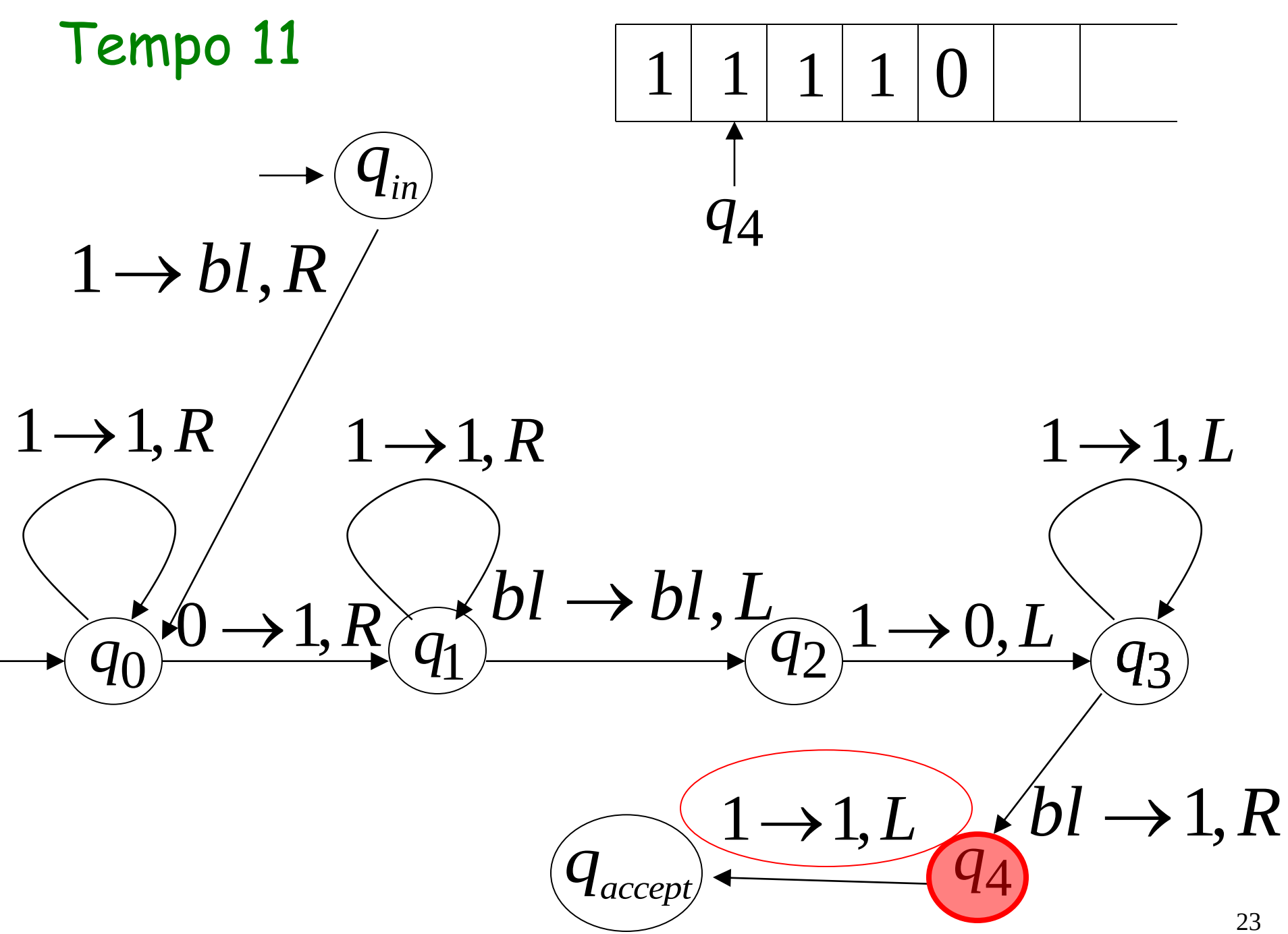
Tempo 9


1 1 1 0
q_3
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
q_{accept}
q_4
$1 \to 1, L$
$bl \to 1, R$

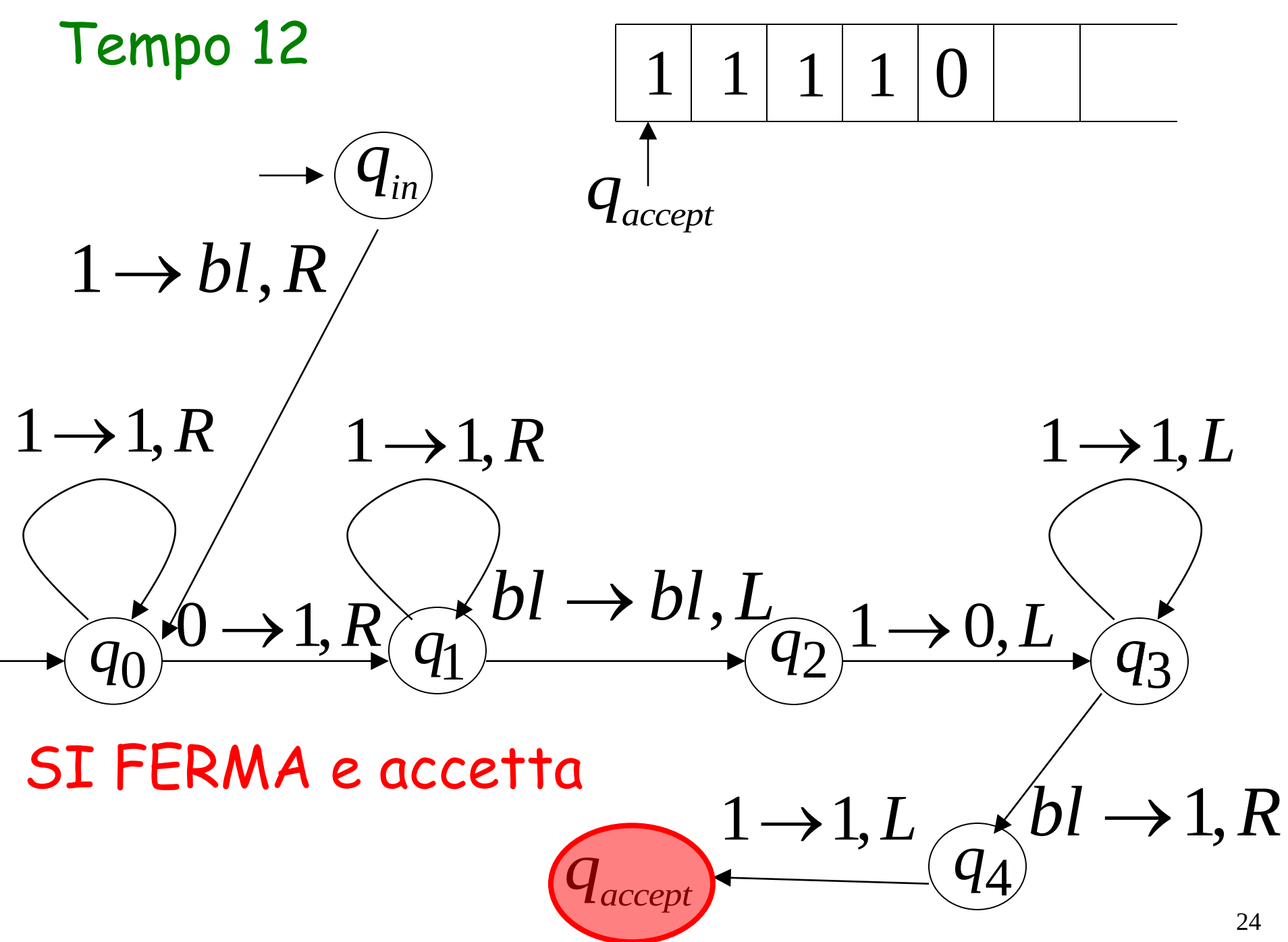
Tempo 10


1 1 1 0
q_3
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
$bl \to 1, R$
$1 \to 1, L$
q_{accept}
q_4

Tempo 11


1 1 1 1 0
q_4
q_{in}
$1 \to bl, R$
$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$bl \to bl, L$
$1 \to 0, L$
q_0
q_1
q_2
q_3
$1 \to 1, L$
$bl \to 1, R$
q_{accept}
q_4

Tempo 12



SI FERMA e accetta

Un altro esempio

La funzione $f(x) = 2x$ e' calcolabile

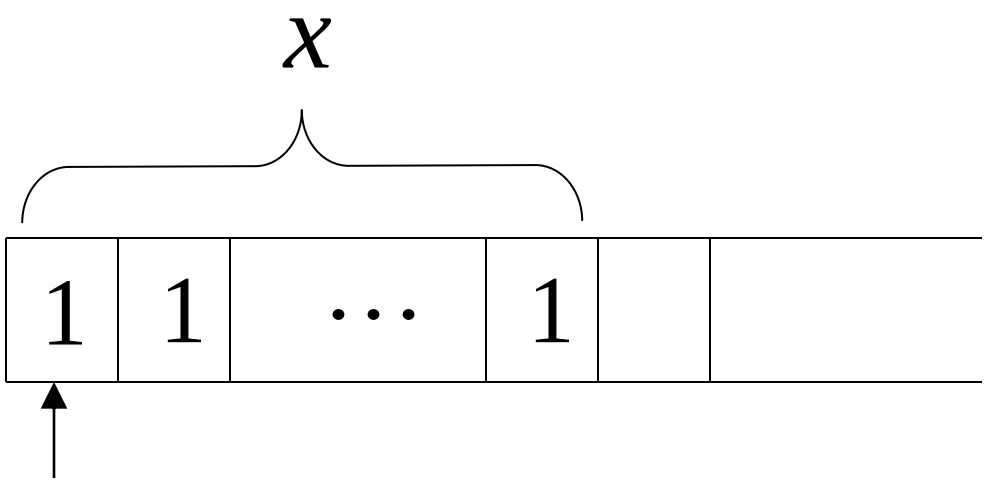
x e' un intero

Macchina di Turing:

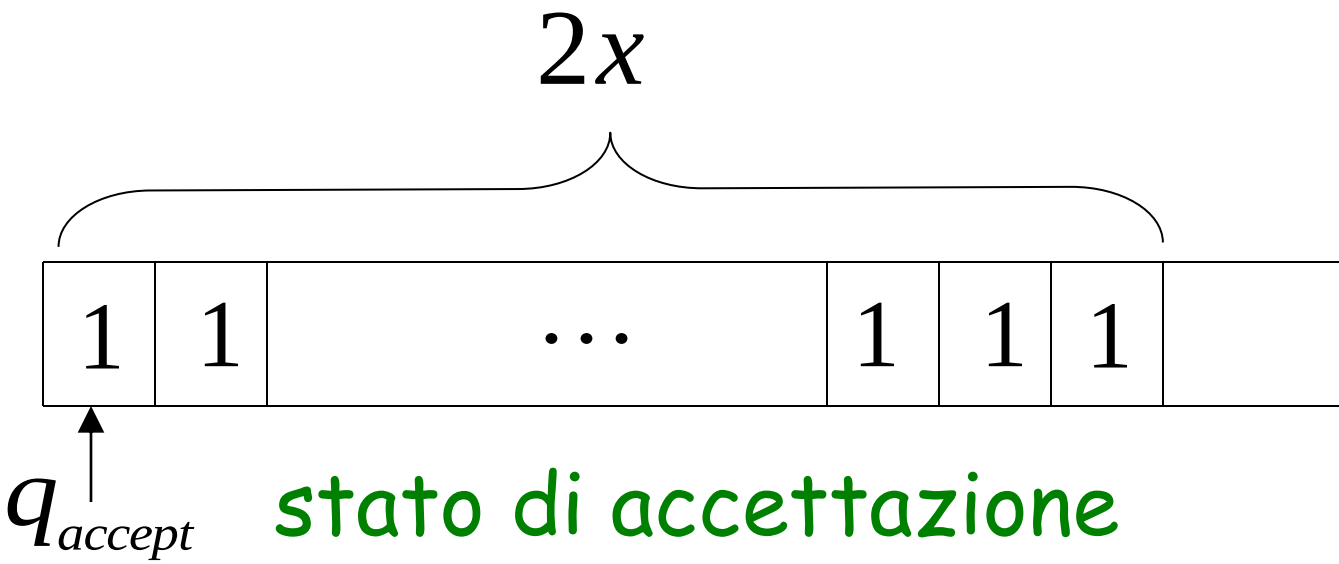
Stringa input: x in notazione unaria

Stringa output: xx in notazione unaria

Inizio


x
1
1
…
1
q_0
stato iniziale

Fine


2x
1
1
…
1
1
1
q_{accept}
stato di accettazione

Un altro esempio

La funzione

e' calcolabile

$$f(x,y) = \begin{cases} 1 & \text{se } x > y \\ 0 & \text{se } x \leq y \end{cases}$$