

Implementare model-checkers
con fixed-point calculus
(fondamenti tool Getafix)

Implementing a model-checker

- In standard programming languages (e.g., C or Java) it is a complex task
 - memory management
 - caching management
 - variable ordering (BDD)
 -
- A typical model checker spans over thousand lines of code
- Small changes in the algorithms may require redesigning large portions of code
 - usually hard to try new ideas

Topics

- A framework to implement model-checking algorithms for Boolean programs (tool GETAFIX)
- Efficient
 - competitive with mature model-checkers
- Easy
 - to implement and debug
 - to experiment with variants
- Amenable to “theory people”
 - high level formalism (fixed-point calculus)
 - hide details unrelated to the algorithmic aspects of solutions

How model-checkers look in our formalism

```
mu bool Reachable( Module s_mod, PrCount s_pc, Local s_CL, s_CL ~+ s_ENTR
s_mod < s_pc, s_pc < s_CL, s_CL ~+ s_ENTR
( ( exists Module t_mod, PrCount t_pc, Local t_CL, t_CL ~+ t_ENTR
  ( target(t_mod,t_pc) & Reachable(t_mod,t_pc,t_CL,t_CL ~+ t_ENTR

| (enforce(s_mod, s_CL, s_CG) & (
  (s_mod=0 & s_pc=0)
  | ( s_pc=0 & CopyLocals(s_mod,s_ENTR
    & (exists Module t_mod, PrCount t_pc, Local t_CL, t_CL ~+ t_ENTR
      ( (Reachable(t_mod,t_pc,t_CL,t_CL ~+ t_ENTR
        & CopyGlobals(s_mod, t_CG, s_ENTR

| (exists PrCount t_pc, Local t_CL, Global t_CG.
  ( (Reachable(s_mod,t_pc,t_CL,t_CL ~+ t_ENTR
    & ( programInt(s_mod,t_pc,s_pc,t_CL,s_CL

| (exists PrCount t_pc, Global t_CG, Module u_mod,
  ( exists Local t_CL. ( (Reachable(s_mod,t_pc,t_CL,t_CL ~+ t_ENTR
    & SkipCall(s_mod,t_pc,s_pc)) & programInt(s_mod,t_pc,s_pc,t_CL,s_CL
    & SetReturnTS(s_mod,u_mod,t_pc,u_pc)
    & ( exists Local u_CL, Global u_CG.
      ( (Reachable(u_mod,u_pc,u_CL,u_CL ~+ u_ENTR
        & SetReturnUS(s_mod,u_mod,t_pc,u_pc)
    )));

( exists Module s_mod, PrCount s_pc, Local s_CL, s_CL ~+ s_ENTR
  ( target(s_mod,s_pc) & Reachable(s_mod,s_pc,t_CL,t_CL ~+ t_ENTR
```

- Checks whether an error state in Boolean program is reachable.
- Entire model checking algorithm in 1~2 pages !
- Symbolic algorithm that uses BDDs
- Competitive with mature model-checkers

- High-level declarative algorithm at one level; but also algorithmic aspects encoded!
- Highly readable ; easily debuggable
- Easy to experiment with variants
- Underlying solver GETAFIX will convert the program and run this algorithm efficiently

Why fixed-point calculus?

- Natural formalism in verification
 - Most symbolic model-checking algorithms essentially compute fixed-points
 - Ex: compute the least X s.t. $X = \text{Start} \vee X \vee \text{Succ}(X)$
(forward reachability)
- Right primitives
 - easy encoding of model-checking algorithms
 - sufficiently low-level to express algorithmic details

Fixed-point calculus

Quantified Boolean logic

First-order logic over the Boolean domain

$$\text{BoolExp } B ::= T \mid F \mid R_k(x_1, \dots, x_k) \mid \neg B \mid \\ B \wedge B \mid B \vee B \mid \exists x.(B) \mid \forall x.(B)$$

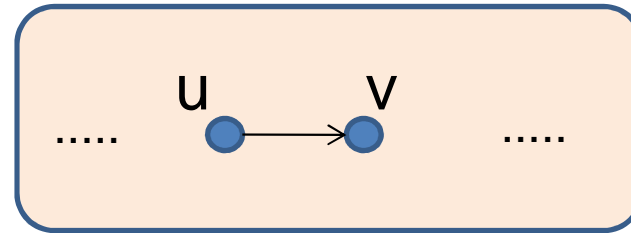
Variables interpreted over the Boolean domain $\{T, F\}$

Relations interpreted as k-ary tuples of $\{T, F\}$

Some relations defined by pgms

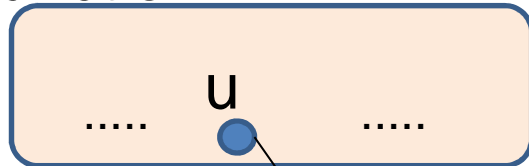
- $\text{Internal}(u,v)$

function f



- $\text{Call}(u,v)$

function f



call g

function g

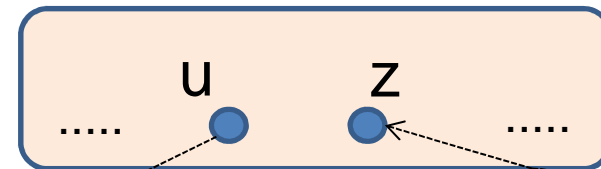
v

Entry

.....

- $\text{Return}(u,v,z)$

function f



call g

function g

v

return

Exit

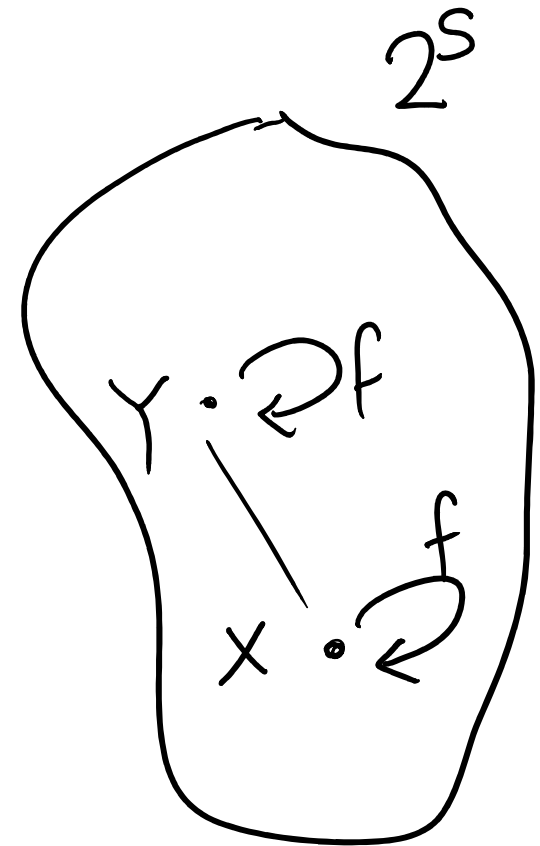
.....

Fixed-points

Consider the complete lattice $(2^S, \subseteq)$

Consider a function $f : 2^S \rightarrow 2^S$

- $X \in 2^S$ is a **fixed point** of f
if $f(X)=X$
- A fixed point X of f is the **least fixed point**
if for every fixed point Y , $Y \supseteq X$



Tarski-Knaster theorem

Thm.

Let L be a complete lattice and let $f : L \rightarrow L$ be a monotonic function.

Then the set of fixed points of f in L is also a complete lattice.

(The theorem was shown just for the powerset lattice earlier by Tarski)

Tarski theorem applied to $(2^S, \subseteq)$

Consider a function $f : 2^S \rightarrow 2^S$ that is monotonic, i.e.:

$$X \subseteq Y \text{ then } f(X) \subseteq f(Y)$$

Consider the sequence

$$X_0 = \emptyset$$

$$X_1 = f(X_0)$$

$$X_2 = f(X_1) \dots$$

*This eventually converges to the least-fixed point
(and hence always exists).*

Es.: Reachability in non-rec programs

Reach(pc, x)

$$= (pc=0 \wedge \text{Init}(x)) \vee \exists pc', y. (\text{Reach}(pc', y) \wedge \text{Internal}(pc', y, pc, x))$$

Declarative: Reach is the smallest set that contains the initial state and is closed under internal image.

Operational/Algorithmic:

Start with the empty relation; keep applying Reach to the current definition till the least fixed-point is reached

Note: Init and Internal are relations that are defined by the program being checked. Algorithm uses these relations.

A fixed-point calculus

- System of equations

$$f_1(x_1, \dots, x_r) = \text{PosBoolExp using } f_1, \dots, f_m$$

$$f_2(y_1, \dots, y_s) = \text{PosBoolExp using } f_1, \dots, f_m$$

\vdots

$$f_m(z_1, \dots, z_t) = \text{PosBoolExp using } f_1, \dots, f_m$$

- Positive Boolean expression: relations do not occur within an odd number of negations
- A positive Boolean expression defines a monotonic function.
- Least fixed-point of all these relations f_1, \dots, f_m exists.

Computation of the least fixed-points

- Least fixed-point of f_1, \dots, f_m can be computed by an algorithm that starts with the empty relation, and computes iteratively till a fixed-point is reached

Let $f=B$ be an equation of a system Eq

Evaluate(f, Eq) :

Set $S := \emptyset$

while (S does not

- $Eq' := Eq \setminus f =$

- $Eq'' := Eq' [f \leftarrow$

- Let f_1, \dots, f_r of

- $S := \text{evaluation}$

}

return S

- Evaluate** gives operational semantics to our algorithms
 - even when general (non positive)Boolean expressions are used (in this case we are in charge to ensure convergence)

Recursive Boolean programs

(sequential) Boolean programs

- Nondeterministic procedural programs
- Has conditionals, iteration, recursive function calls
- Variables range over the Boolean domain {True, False}
- And specific statements for:
 - function invariants (enforce)
 - assertions (assume)
 - constrained assignments (constrain)

A Boolean program

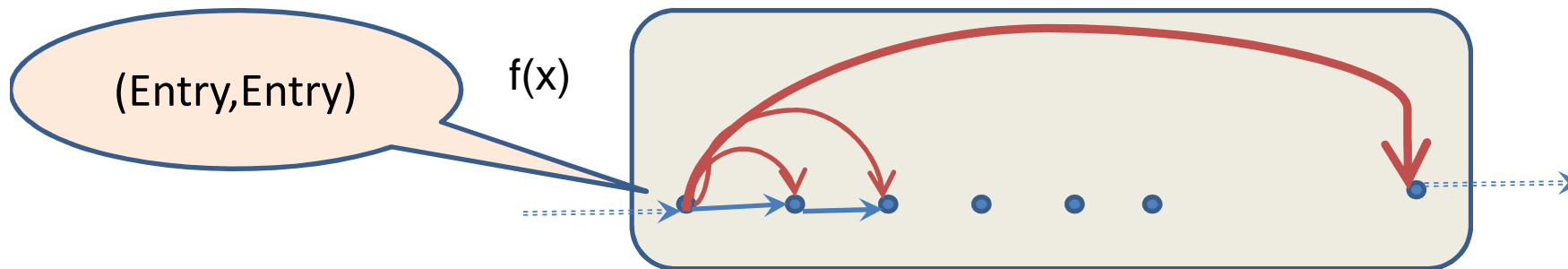
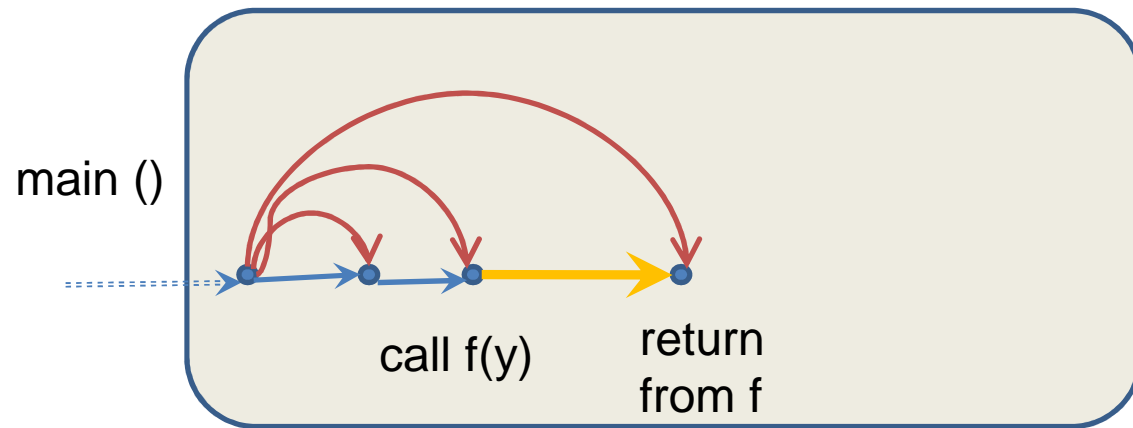
```
main () {  
  bool a,b;  
  a := T;  
  b := *;  
  while (b) {  
    a:= f(b);  
    b := * constrain (!(a && b'));  
  }  
  if (c) {  
    ERR: skip  
  }  
}
```

Nondeterminism:
"b is either T or F"

```
bool f( bool x ) {  
  bool y;  
  y := *;  
  while (y) {  
    y:= f(x)  
  }  
  return y;  
}
```

Constrained assignment
"b is F" or "a is F"

Checking recursive Boolean programs



Summaries: (Entry, State)

meaning the state is reachable from the entry of the function (perhaps using calls to other functions)

Writing the algorithm

$Summary(u, v) =$

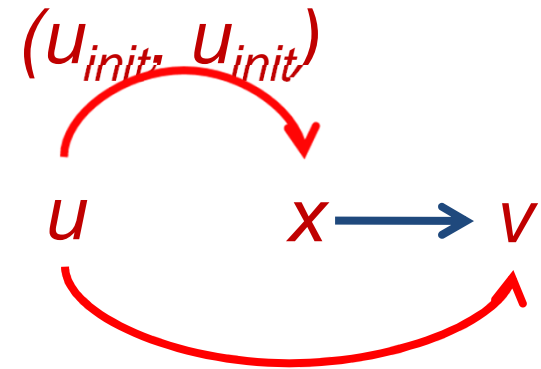
$(Entry(u.pc) \wedge u=v \wedge Init(u.pc))$

$\vee \exists x. (Summary(u, x) \wedge Internal(x, v))$

$\vee \exists x, y. (Summary(x, y) \wedge Call(y, u) \wedge u=v)$

$\vee \exists x, y, z. (Summary(u, x) \wedge Call(x, y)$

$\wedge Summary(y, z) \wedge Exit(z.pc) \wedge Return(x, z, v)$)



Writing the algorithm

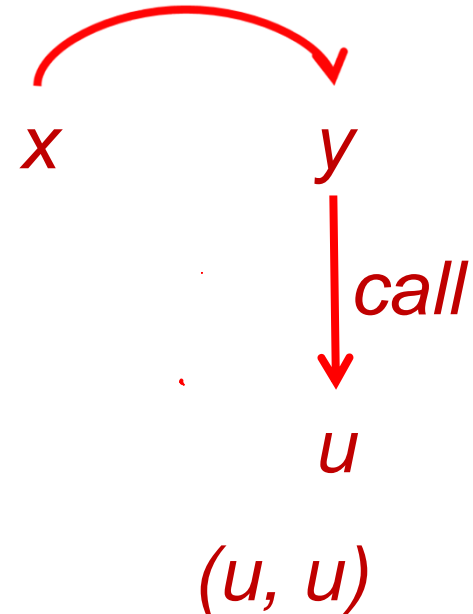
$Summary(u, v) =$

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$\vee \exists x. (Summary(u, x) \wedge Internal(x, v))$

$\vee \exists x, y. (Summary(x, y) \wedge Call(y, u) \wedge u=v)$

$\vee \exists x, y, z. (Summary(u, x) \wedge Call(x, y) \wedge Summary(y, z) \wedge Exit(z.pc) \wedge Return(x, z, v))$



Writing the algorithm

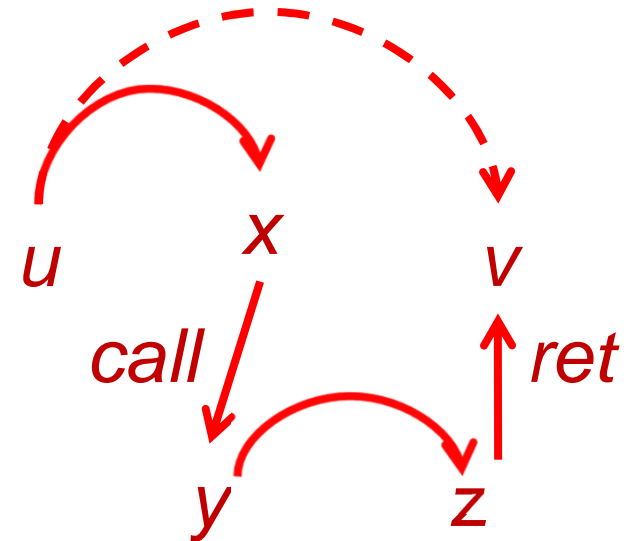
$Summary(u, v) =$

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$\vee \exists x, y. (Summary(x, y) \wedge Call(y, u) \wedge u=v)$

$\vee \exists x, y, z. (Summary(u, x) \wedge Call(x, y)$
 $\wedge Summary(y, z) \wedge Exit(z.pc) \wedge Return(x, z, v)$
 $)$



Actual code

```
mu bool Reachable( Module s_mod, PrCount s_pc, Local s_CL, Global s_CG, Local s_ENTRY_CL, Global
s_ENTRY_CG)
s_mod < s_pc, s_pc < s_CL, s_CL ~+ s_ENTRY_CL, s_CL < s_CG, s_CG ~+ s_ENTRY_CG /* BDD ordering */
( ( exists Module t_mod, PrCount t_pc, Local t_CL, Global t_CG, Local t_ENTRY_CL, Global t_ENTRY_CG.
    ( target(t_mod,t_pc) & Reachable(t_mod,t_pc,t_CL,t_CG,t_ENTRY_CL,t_ENTRY_CG) ) ) )

| (enforce(s_mod, s_CL, s_CG) & (
    (s_mod=0 & s_pc=0 )
    | ( s_pc=0 & CopyLocals(s_mod,s_ENTRY_CL,s_CL)
        & (exists Module t_mod, PrCount t_pc, Local t_CL, Global t_CG, Local t_ENTRY_CL, Global t_ENTRY_CG.
            ( (Reachable(t_mod,t_pc,t_CL,t_CG,t_ENTRY_CL,t_ENTRY_CG)
                & CopyGlobals(s_mod, t_CG, s_ENTRY_CG)
                & programInt(s_mod,t_pc,s_pc,t_CL,s_CL,t_CG)
            )
        )
    | (exists PrCount t_pc, Local t_CL, Global t_CG.
        ( (Reachable(s_mod,t_pc,t_CL,t_CG,s_ENTRY_CL,s_ENTRY_CG)
            & ( programInt(s_mod,t_pc,s_pc,t_CL,s_CL,t_CG)
                & SkipCall(s_mod,t_pc,s_pc) & programCall(s_mod,t_pc,s_pc,t_CL,s_CL,t_CG)
                & SetReturnTS(s_mod,u_mod,t_pc,u_pc,t_CL,s_CL,s_CG)
            )
        )
    | (exists PrCount t_pc, Global t_CG, Module u_mod, Local u_PC, Local u_CL, Global u_CG.
        ( exists Local t_CL. ( (Reachable(s_mod,t_pc,t_CL,t_CG,s_ENTRY_CL,s_ENTRY_CG)
            & SkipCall(s_mod,t_pc,s_pc) & programCall(s_mod,t_pc,s_pc,t_CL,s_CL,t_CG)
            & SetReturnTS(s_mod,u_mod,t_pc,u_pc,t_CL,s_CL,s_CG)
        )
        & ( exists Local u_CL, Global u_CG.
            ( (Reachable(u_mod,u_pc,u_CL,u_CG,u_ENTRY_CL,u_ENTRY_CG)
                & SetReturnUS(s_mod,u_mod,t_pc,u_pc,u_CL,s_CL,s_CG)
            )
        )
    )
));

( exists Module s_mod, PrCount s_pc, Local s_CL, Global s_CG, Local s_ENTRY_CL, Global s_ENTRY_CG.
    ( target(s_mod,s_pc) & Reachable(s_mod,s_pc,s_CL,s_CG,s_ENTRY_CL,s_ENTRY_CG) ) ) )
```

*Code is executed according to
the fixed-point algorithm*
- *bracketing is respected in the
evaluations*

*Proper bracketing can reduce the
number of variables in the
intermediate BDDs*

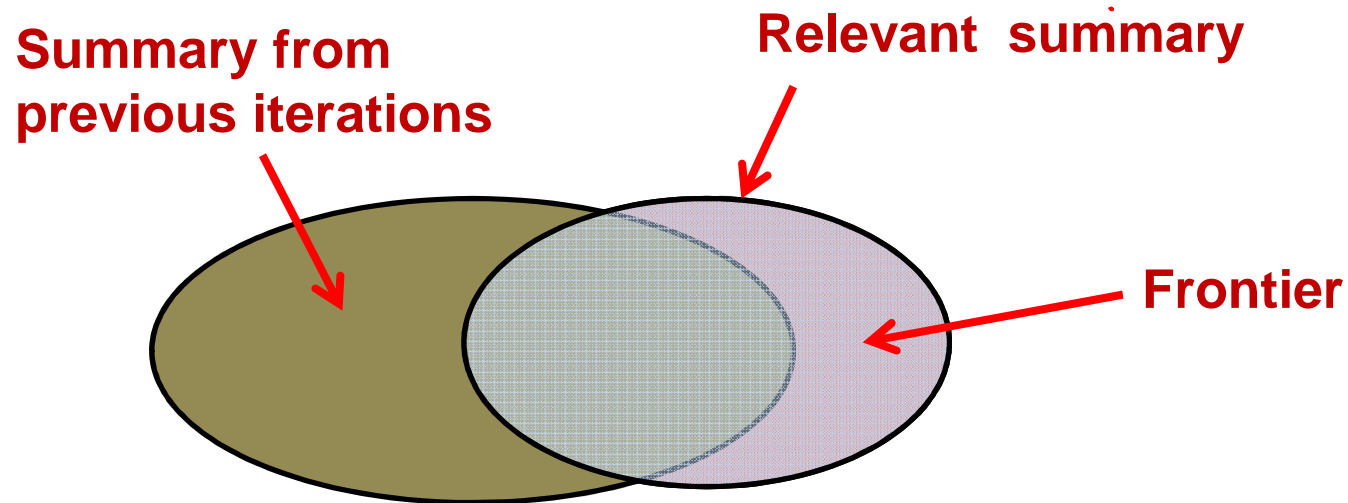
Correctness of the algorithm

- Let P be a recursive Boolean program
- For each pair (u,v) which is added to Summary
 - u is either an initial state or a reachable entry
 - v is either u or (u,x) is in Summary and v is reachable from x by an internal move or by a call which returns(only reachable states are added to Summary)
- Each reachable state is eventually added to Summary
- Theorem.
A state v is reachable in P if and only if
$$\exists \text{ an entry } u \text{ such that } \text{Summary}(u,v)$$

A simpler summary-based algorithm

- Modify Summary such that
 - clause $\exists x, y. (Summary(x, y) \wedge Call(y, u) \wedge u=v)$ is deleted
 - clause $(Entry(u.pc) \wedge u=v \wedge Init(u.pc))$ is replaced by $(Entry(u.pc) \wedge u=v)$
- Still answers reachability
- May also compute unreachable states

An optimized algorithm



Frontier: Newly discovered summaries in the last round

Relevant: Summaries whose program counter value is involved in the frontier

Idea: Compute only on the relevant summaries.

An optimized algorithm

- Why not computing simply on Frontier?
 - BDDs for Frontier can be larger than reachable set
 - Relevant is a restriction of the reachable set to a particular set of program counters
- New Frontier is computed into two steps:
 - New1: image-closure of Relevant on internal transitions
 - New2: image of Relevant on calling a module or skipping the call using a summary
- Handling call and return transitions is expensive compared to internal transitions
- Programs contain many more internal than other transitions

Writing the optimized algorithm

- $\text{Summary}(1, u, v)$ denote the computed summaries (u, v)
- $\text{Summary}(0, u, v)$ denote the summaries (u, v) computed before the last round
 - (u, v) is in Frontier if $\text{Summary}(1, u, v)$ and not $\text{Summary}(0, u, v)$

- Transitive closure on internal transitions:

$$\text{New1}(u, v) = (\text{Summary}(1, u, v) \wedge \text{Relevant}(v.pc))$$

$$\vee (\exists x. (\text{New1}(u, x) \wedge \text{ProgramInt}(x, v)))$$

Writing the optimized algorithm

$$\begin{aligned} \text{New2}(u, v) = & (\exists x. (\text{Relevant}(x.pc) \wedge \text{Summary}(1, u, x) \wedge \text{Call}(x, v))) \\ & \vee (\exists x, y, z. (\text{Summary}(u, x) \\ & \quad \wedge \text{IntoCall}(x, y) \wedge \text{Summary}(y, z) \\ & \quad \wedge \text{Exit}(z.pc) \wedge \text{Return}(x, z, v) \\ & \quad \wedge (\text{Relevant}(x.pc) \vee \text{Relevant}(z.pc)))) \end{aligned}$$

- Compute calls and returns on relevant program counters
- Note: either the caller or the exit are required to be relevant to jump from a caller to a matching return

Writing the optimized algorithm

$$\begin{aligned} \text{Summary}(fr, u, v) = & (fr=1 \wedge \text{Entry}(u.pc) \wedge u=v \wedge \text{Init}(u.pc)) \\ & \vee \text{Summary}(1, u, v) \\ & \vee (fr=1 \wedge (\text{New1}(u, v) \vee \text{New2}(u, v))) \end{aligned}$$

- Updates the already computed summaries with the current Frontier
- Adds new summaries as Frontier

$$\begin{aligned} \text{Relevant}(pc) = & \exists u, v. (\text{Summary}(1, u, v) \\ & \wedge \neg \text{Summary}(0, u, v) \wedge v.pc=pc) \end{aligned}$$

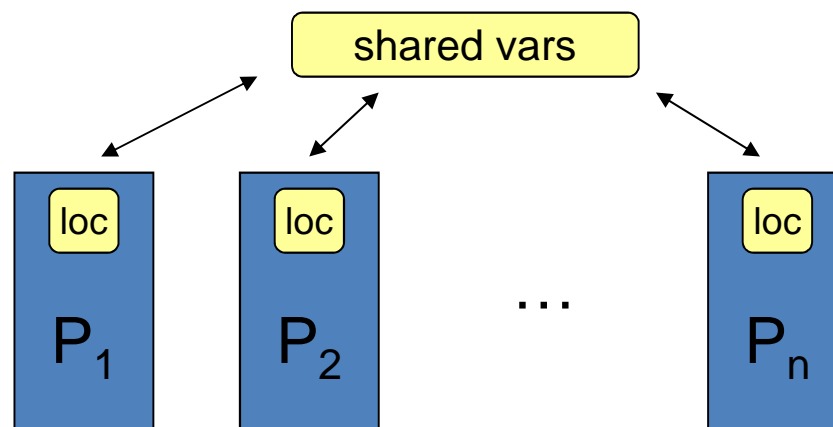
Problem

- Relevant does not grow monotonically!
- Tarski-Knaster theorem does not apply
- Convergence of the algorithm is up to you
 - assume the algorithmic semantics to compute the least fixed-point

Concurrent Boolean programs

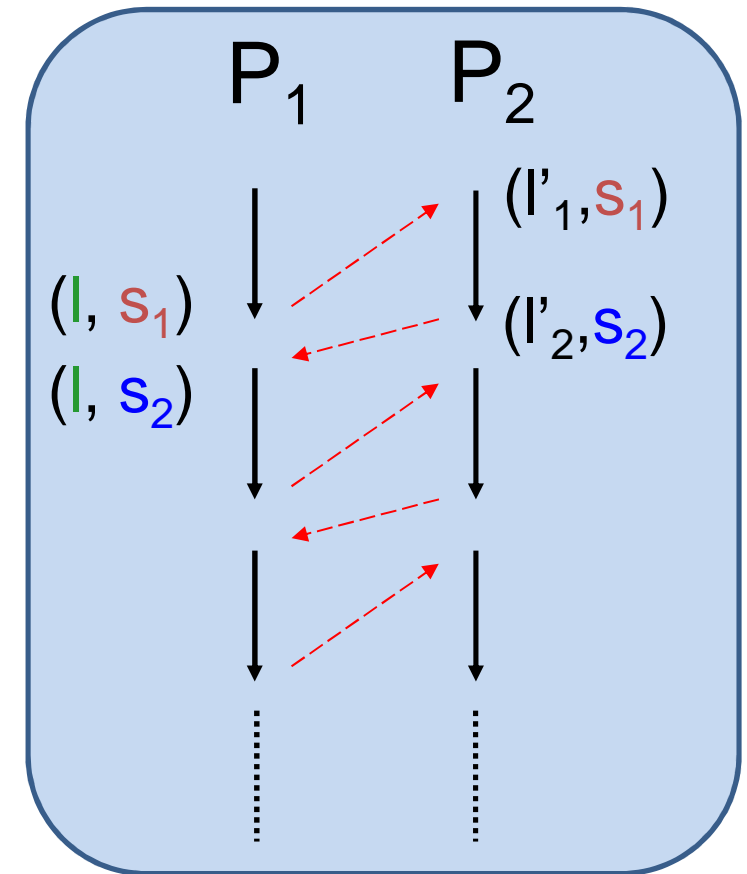
Concurrent Boolean programs

- A fixed number of recursive Boolean programs P_1, \dots, P_n (running in parallel)
- each program has its own local variables
 - local to the program
- communication is through shared variables



Concurrent Boolean programs

- Global states (l, g)
 - g is a valuation of shared variables (*shared state*)
 - l is *local state*
- Semantics by interleaving:
 - computations as sequences of execution contexts
 - only one P_i is active in each context
 - either the active P_i moves (*local behavior*)
 - or control switches to another component (*context-switch*)



Reachability of concurrent Boolean prgms

Given a concurrent Boolean program and a particular position of a component,

is that position reachable?

Problem is **UNDECIDABLE !**

(two recursive Boolean programs sharing finitely many Boolean variables can simulate a Turing machine)

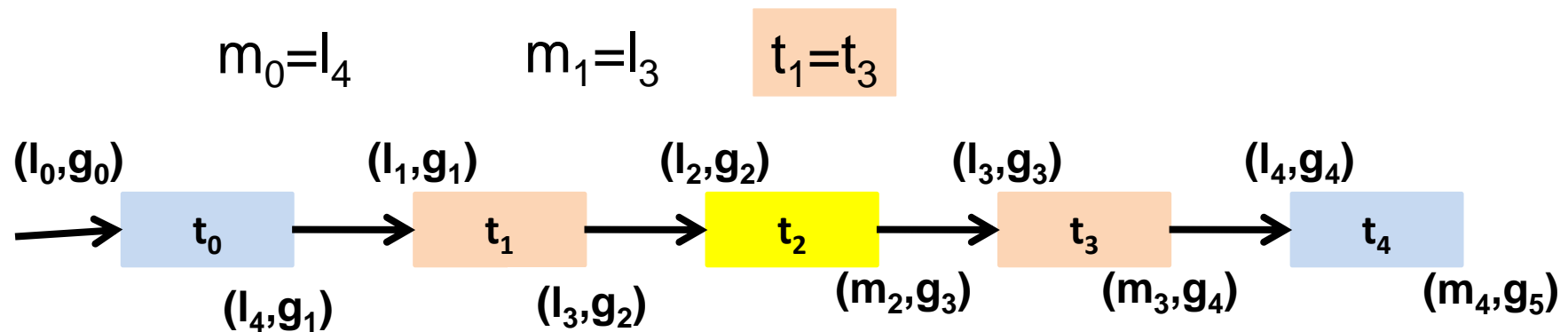
Bounded context-switching

Fix k .

Is an error reachable within k context-switches?

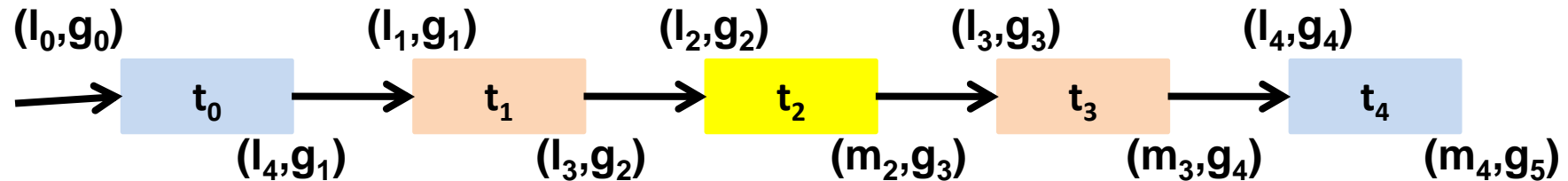
- [Qadeer-Rehof, TACAS'05]: Decidable.
 - Proof uses tuples of automata that capture stack contents of processes
 - Each such automaton is grown as for sequential programs [Schwoon, PhdThesis 2000][Esparza-Schoon, CAV'01]
- Most concurrency-related bugs often show up within few contexts [Musuvathi-Qadeer, PLDI'07]

A computation up to 4 context-switches



- t_0, \dots, t_4 = active component in each context (colors identify components)
- l_0, \dots, l_4 = local states at the beginning of each context
- m_0, \dots, m_4 = local states at the end of each context
- a re-activated component restarts from the last visited local state (when last context-switched out)
- g_1, \dots, g_4 = shared states at context-switches

Towards a summary relation



- Consider a summary (u, v) as for sequential prgms
 - add summaries within the same context is fine
- Suppose $v = (m_2, g_3)$, we want to add a *consistent* summary (u', v') where $v' = (l_3, g_3)$
- We can add (u', v') if we know that according to a global run matching g_0, \dots, g_5 and t_0, \dots, t_5
 - (u, v) was added in context 2 and
 - (u', v'') , $v'' = (l_3, g_2)$, was added in context 1

A summary relation

- Summary tuples are of the form:

$$(u, v, ecs, cs, \{g_i\}_{i=1,..k}, \{t_i\}_{i=0,..k})$$

meaning that there is a global computation which

- visits an entry state **u** in context **ecs** and then **v** in context **cs**
- **u** and **v** are in the same function of program $t_{cs}=t_{ecs}$ (and **(u,v)** is a *summary* edge)
- uses **cs** context-switches
- executes **t_i** at each context **i**
- shared state at the i-th context-switch is **g_i**

Fixed-point formulation

- Let $G = \{g_i\}_{i=1, \dots, k}$, $T = \{t_i\}_{i=0, \dots, k}$
- $Reach(u, v, ecs, cs, G, T) = \phi_{init} \vee \phi_{int} \vee \phi_{call} \vee \phi_{ret} \vee \phi_{1st-switch} \vee \phi_{switch}$

$$\phi_{init} = (cs = ecs = 0 \wedge Entry(u.pc) \wedge u = v \wedge Init(t_0, u.pc))$$

$$\phi_{int} = \exists x. (Reach(u, x, ecs, cs, G, T) \wedge ProgramInt(x, v))$$

$$\phi_{call} = \exists x, y, ecs. (Reach(x, y, ecs, cs, G, T) \wedge Call(y, u) \wedge \textcolor{red}{ecs} = cs \wedge u = v)$$

Fixed-point formulation

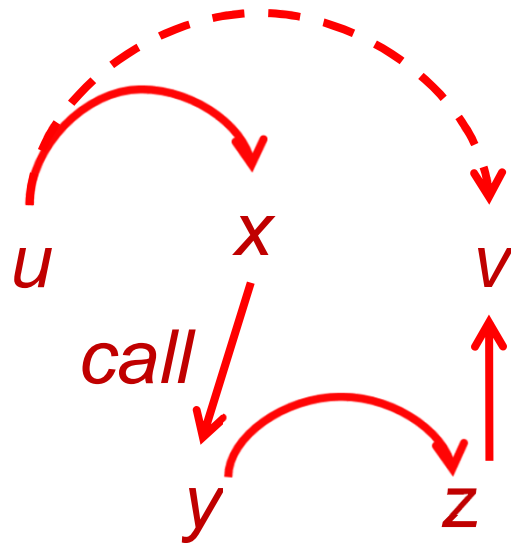
$$\phi_{ret} = \exists x, y, z, cs'.$$

(*Reach*(*u*, *x*, *ecs*, *cs'*, *G*, *T*)

\wedge *Call* (*x*, *y*) \wedge *Reach*(*y*, *z*, *cs'*, *cs*, *G*, *T*)

\wedge *Exit*(*z.pc*) \wedge *Return*(*x*, *z*, *v*) \wedge $cs' \leq cs$

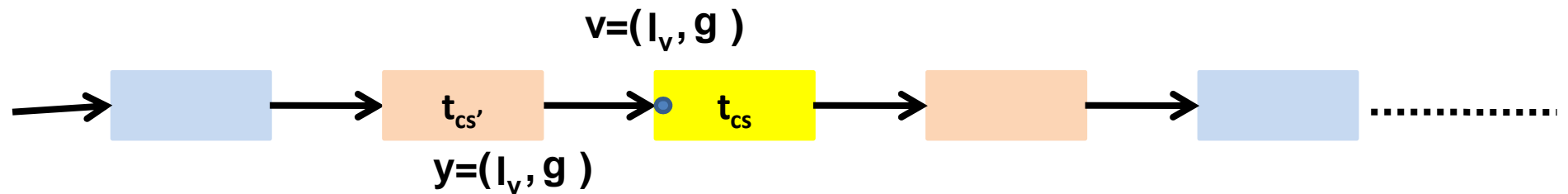
)



Caller's **cs** must
be less than the
cs at return

Fixed-point formulation

$$\begin{aligned}
 \phi_{1st-switch} = & \exists x, y, cs', ecs'. \\
 & (Reach(x, y, ecs', cs', G, T) \\
 & \quad \wedge (cs = cs' + 1) \wedge First(t_{cs}, cs, T) \\
 & \quad \wedge (v.Global = g_{cs} = y.Global) \\
 & \quad \wedge (u = v) \wedge (ecs = cs) \wedge Init(t_{cs}, v.pc) \\
 &)
 \end{aligned}$$



Fixed-point formulation

$\phi_{switch} =$

$(\exists x, y, cs, ecs.$

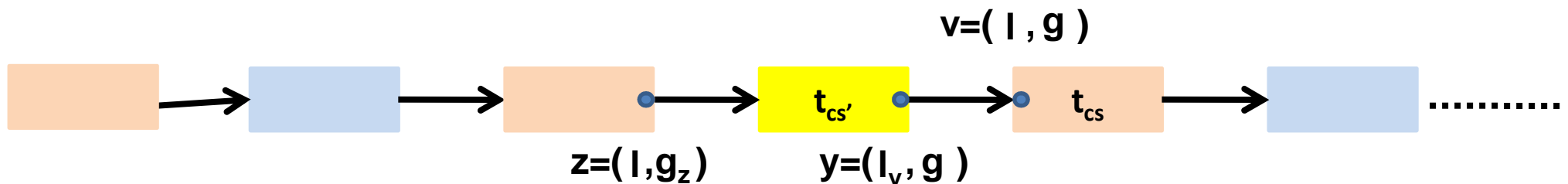
$(Reach(x, y, ecs, cs, G, T) \wedge (cs = cs + 1)$

$\wedge \neg First(t_{cs}, cs, t)$

$\wedge (v.Global = g_{cs} = y.Global)))$

$\wedge (\exists z, cs''. (Reach(u, z, ecs, cs'', G, T) \wedge (cs'' < cs)$

$\wedge Consecutive(cs'', cs, T) \wedge z.Local = v.Local))$



Parameterized Boolean programs

Parameterized Boolean programs

- A fixed number of recursive Boolean programs P_1, \dots, P_n
- A fixed number of shared variables
- Each P_i can be executed on possibly unboundedly many threads
- Each computation has a fixed number of threads
 - threads are not created dynamically
- Interesting class of programs (e.g., device drivers)

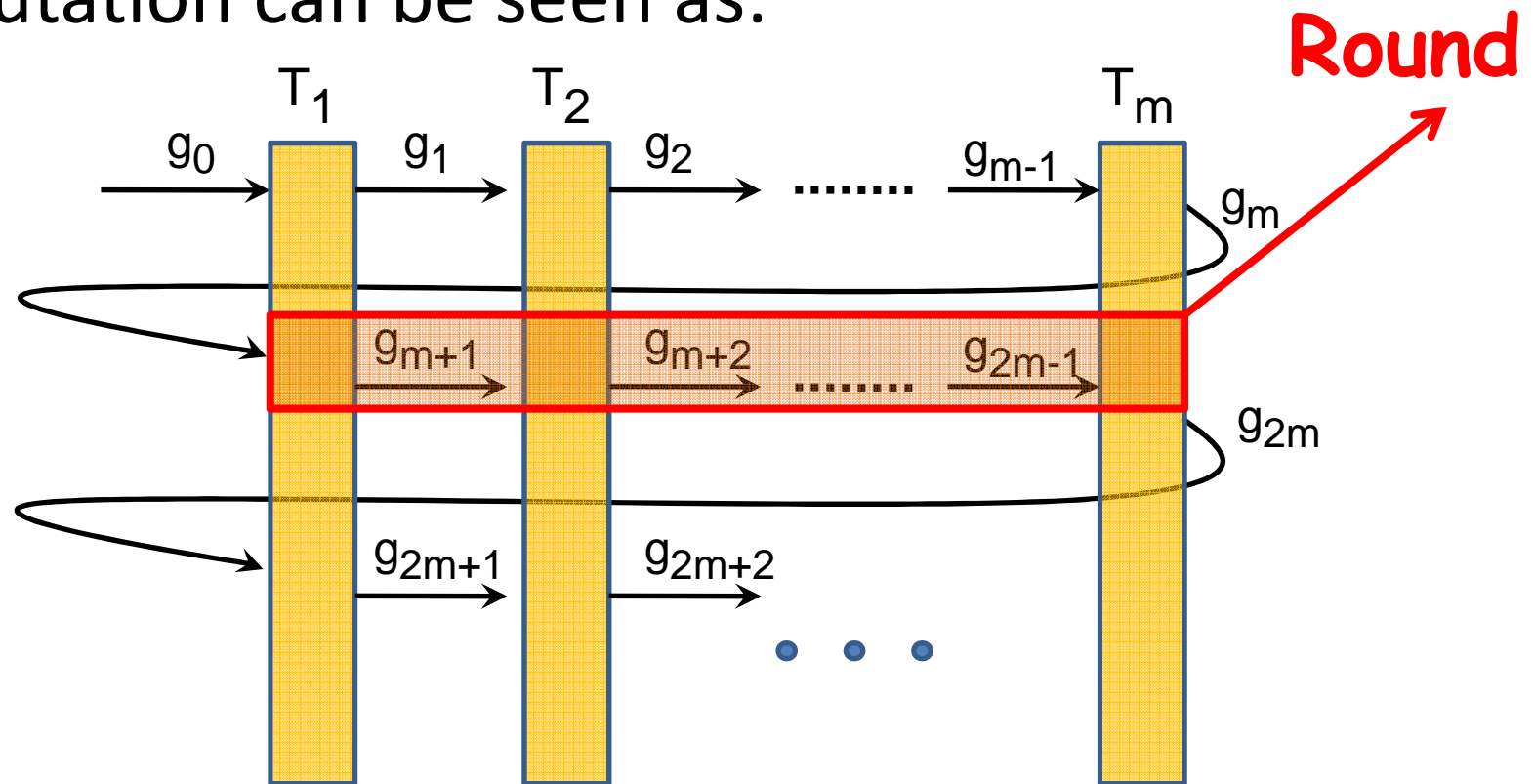
Parameterized Boolean programs

- Infinite number of states:
 - each thread is possibly recursive
 - number of threads is unbounded
- Reachability is clearly undecidable
- What about reachability within a bounded number of context-switches?

Decidable!

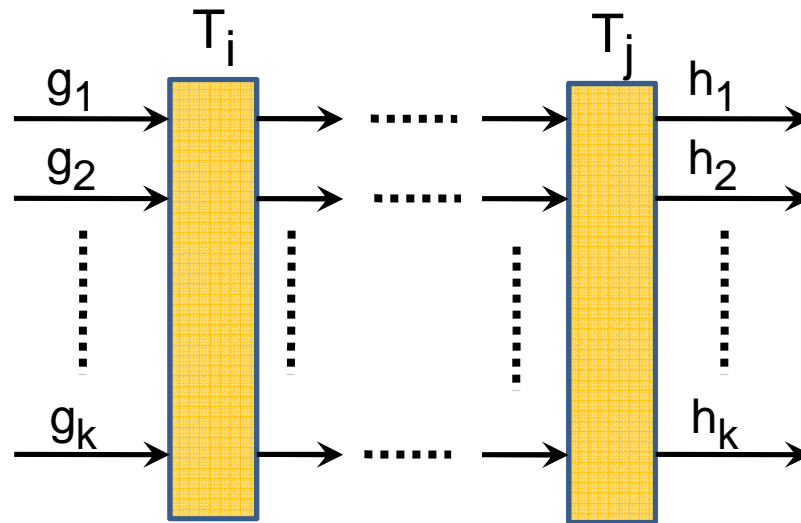
Round-robin scheduling of threads

- Fix a round-robin scheduling
- A computation can be seen as:



Linear Interface

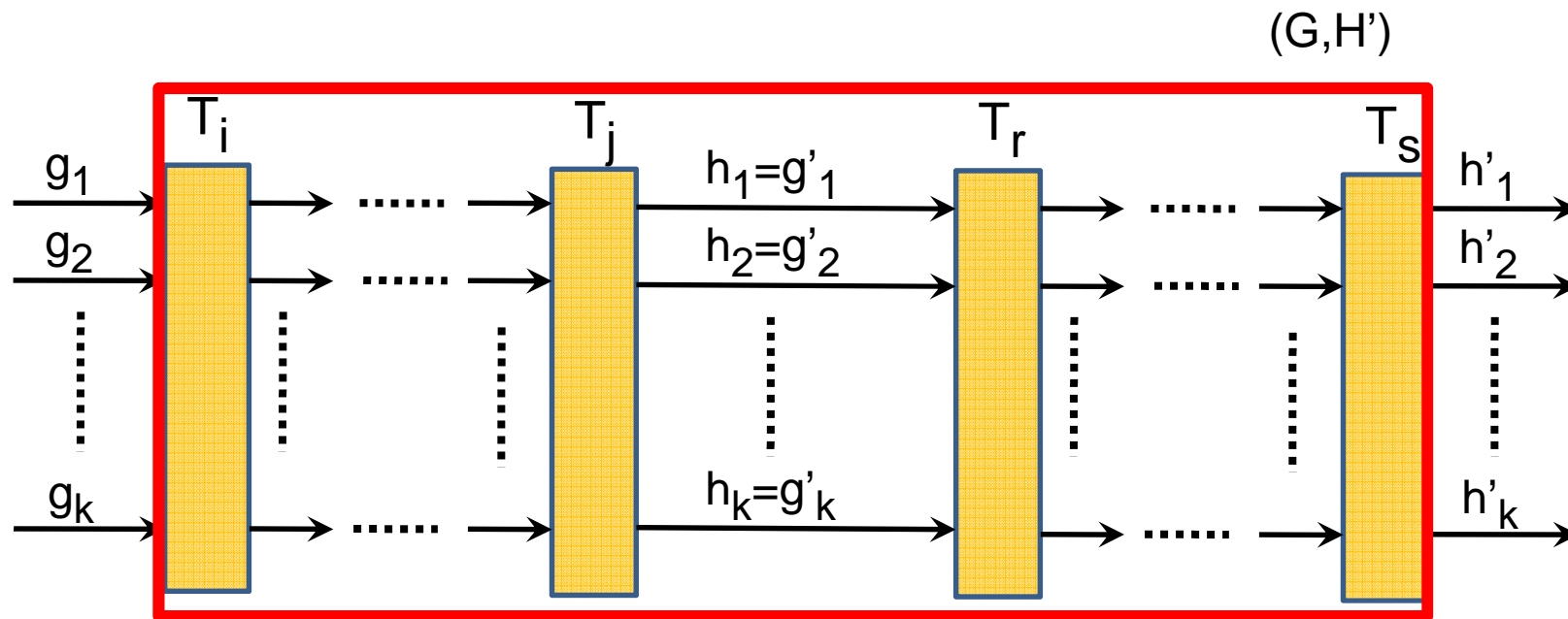
- Let $G=(g_1,\dots,g_k)$ and $H=(h_1,\dots,h_k)$
- (G,H) is a k -linear interface if:



- G is the **input** and H is the **output**

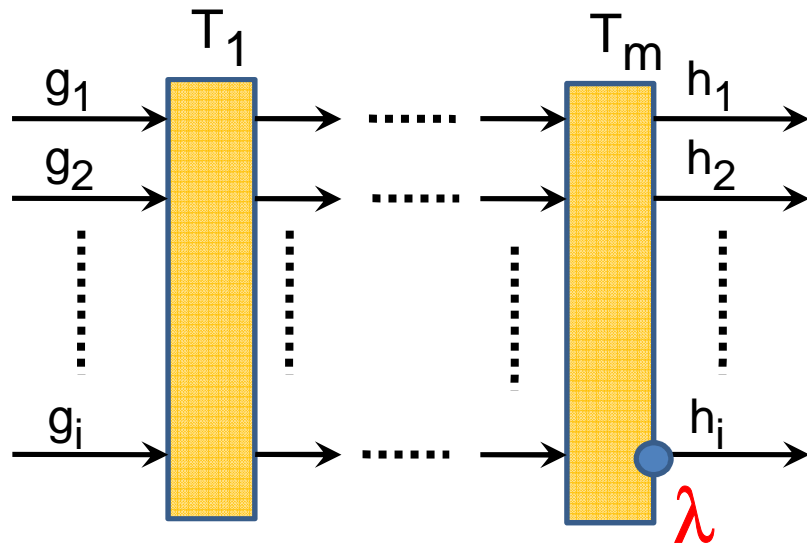
Linear Interfaces compose

- Let (G, H) and (G', H') be linear interfaces s.t. $H = G'$. Then, (G, H') is a linear interface.



A summary relation for parametrized prgms

- Let $G=(g_1,\dots,g_k)$ and $H=(h_1,\dots,h_k)$
- $\text{EagerLI}(i, \lambda, G, H)$ means that
 - (G, H) is a linear interface over threads T_1, \dots, T_m



- λ is a local state
- i is current round
- (λ, h_i) current state

A fixed point-algorithm

EagerLI(i, λ, G, H) =

$(i = 1 \wedge \text{LocalInit}(\lambda) \wedge g_1 = h_1)$

$\vee (i > 1 \wedge g_i = h_i \wedge \text{EagerLI}(i - 1, \lambda, G, H))$

$\vee \langle \text{local reachability} \rangle$

$\vee (\exists G', \lambda'. \text{EagerLI}(i, \lambda', G, G') \wedge \text{EagerLI}(i, \lambda, G', H))$

starts each context



explores each context



composes linear interfaces

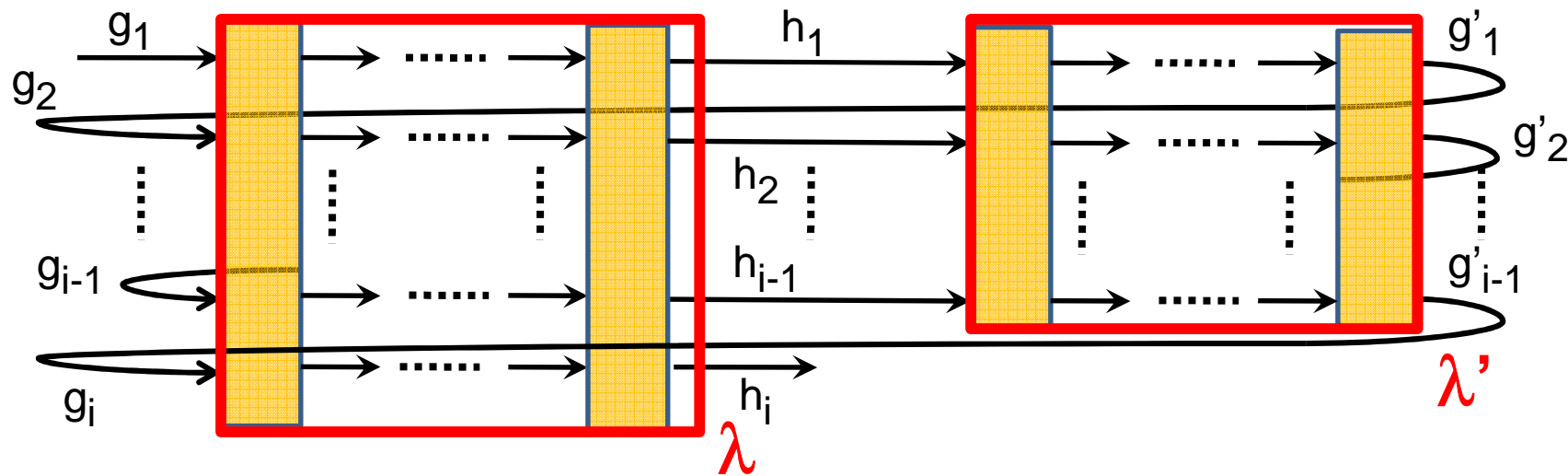


Theorem.

EagerLI is the set of all the linear interfaces of length k for the considered parameterized Boolean program

A fixed point-algorithm

$$\text{Reach}(\lambda, g) = \exists G, H, i. (\text{SharedInit}(g_1) \wedge \text{EagerLI}(i, \lambda, G, H) \wedge g = h_i \\ \wedge (i=1 \vee \exists G', \lambda'. (i > 1 \wedge \text{EagerLI}(i-1, \lambda', H, G') \wedge \text{Wrap}(G, G'))))$$



Theorem.

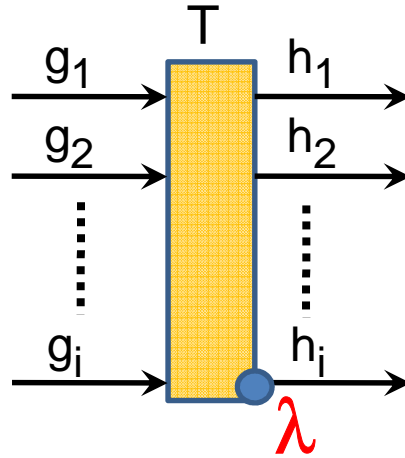
(λ, g) is reachable in P if and only if
 $\text{Reach}(\lambda, g)$

A lazy fixed-point algorithm

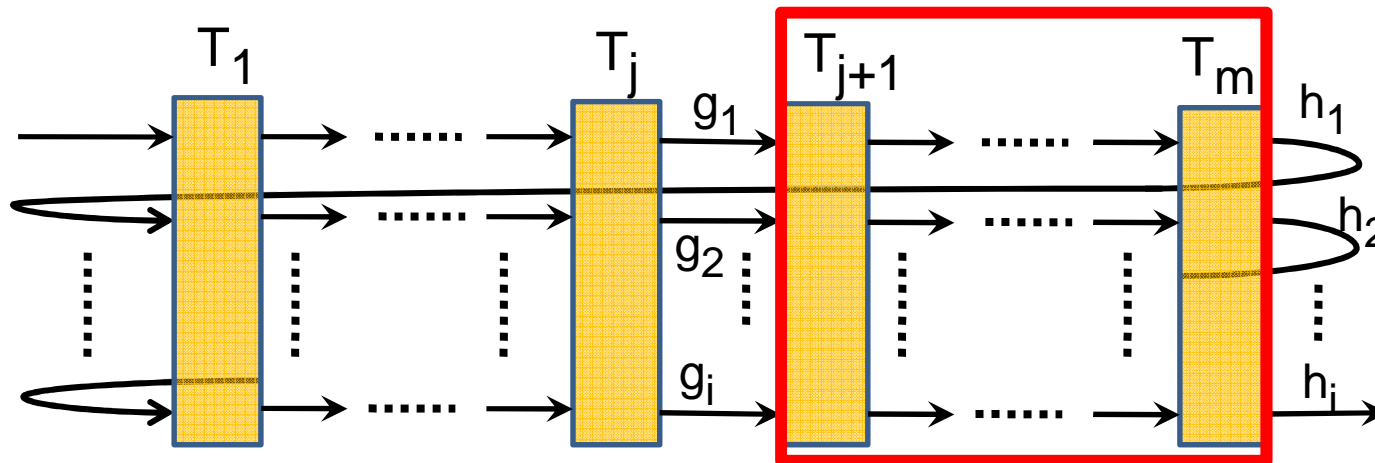
- $\text{Reach}(\lambda, g)$ explores also unreachable states
 - May be inefficient in practice
- We want to compute linear interfaces by exploring only reachable states
- Formula is more involved, we need more relations

Relations

- $\text{ThrdLI}(i, \lambda, G, H)$:

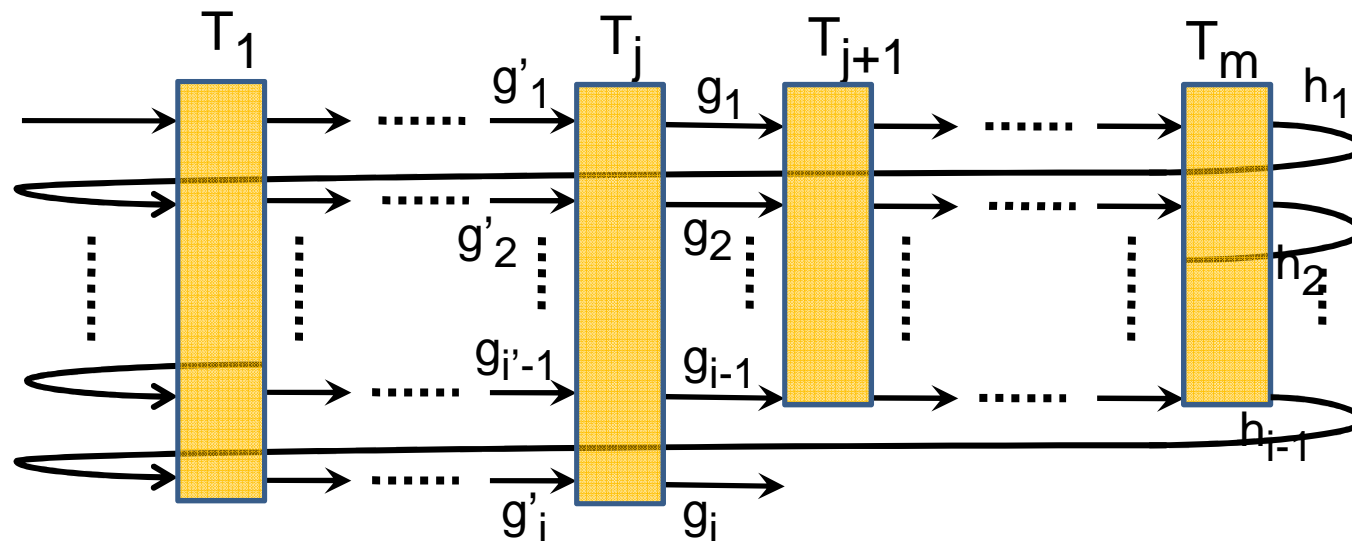


- $\text{RightLI}(i, G, H)$: (G, H) is a right linear interface of size i



Relations

- $\text{WantRightLI}(i, G, H)$ means that there exists a run



- (G, H) is a right linear interface of size $i-1$
- (G', G) is a linear interface of size i over a single thread

Fixed-point formulation

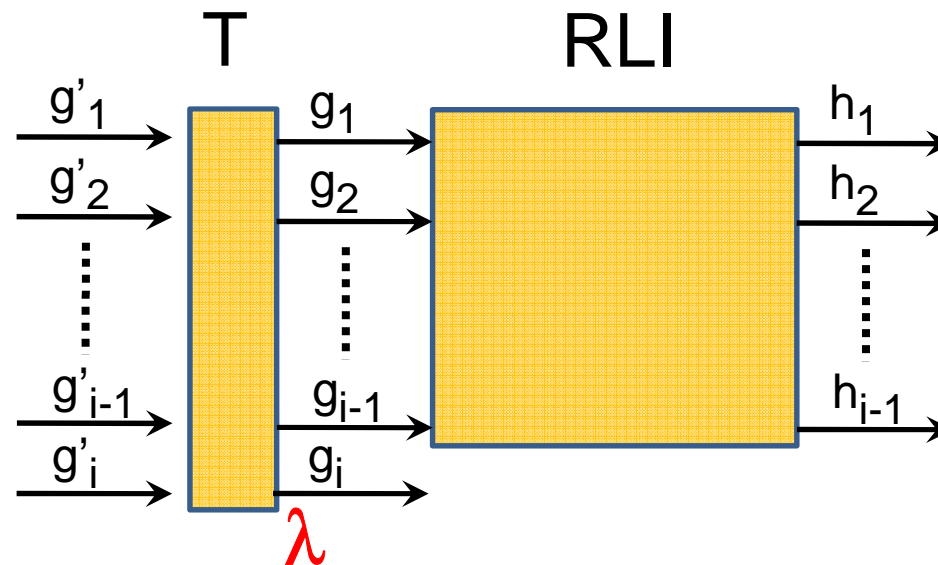
WantRightLI(i, G, H) =

$\exists G', \lambda. (\text{ThrdLI}(i, \lambda, G', G) \wedge \text{RightLI}(i - 1, G, H)$

\wedge *existence of a global run as before*

$(\text{WantRightLI}(i, G', H)$

$\vee (\text{SharedInit}(g'_1) \wedge \text{Wrap}(G', H))))$



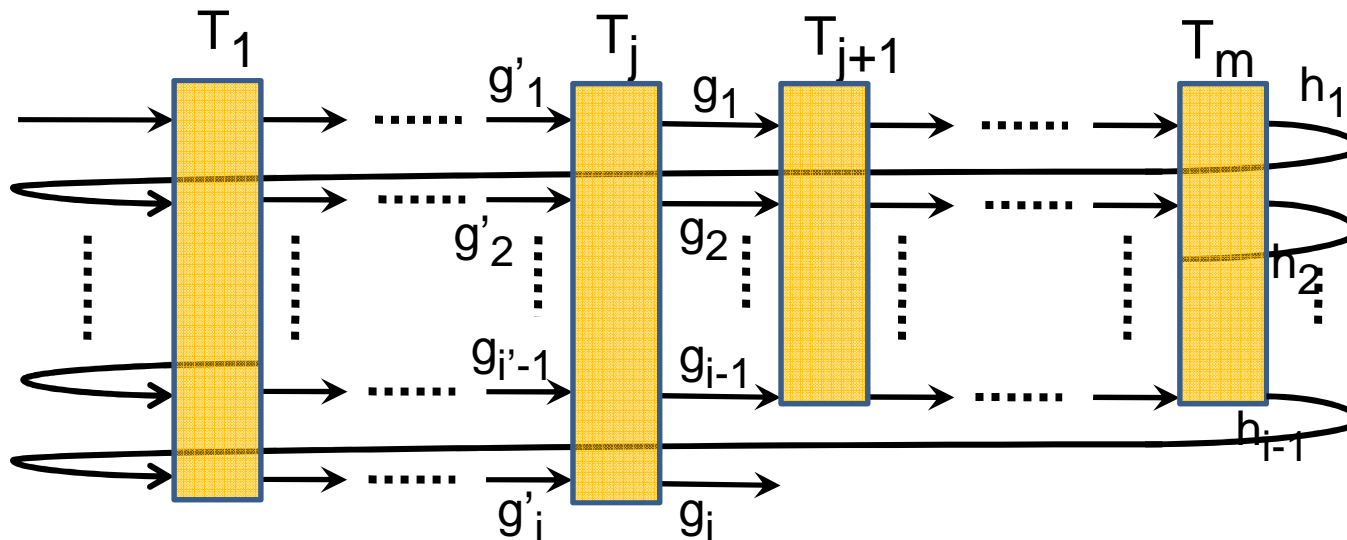
Fixed-point formulation

RightLI(i, G, H) =

$\exists \lambda. (\text{ThrdLI}(1, \lambda, G, H)$

$\vee \exists H'. (\text{ThrdLI}(1, \lambda, G, H') \wedge \text{RightLI}(i, H', H)))$

$\wedge (i = 1 \vee \text{WantRightLI}(i, G, H))$



Fixed-point formulation

$\text{ThrdLI}(i, \lambda, G, H) =$

$(i = 1 \wedge \text{LocalInit}(\lambda) \wedge g_1 = h_1$

**(starts each context
in first round)**

$\wedge (\text{SharedInit}(g_1) \vee \exists G', \lambda'. \text{ThrdLI}(1, \lambda', G', G)))$

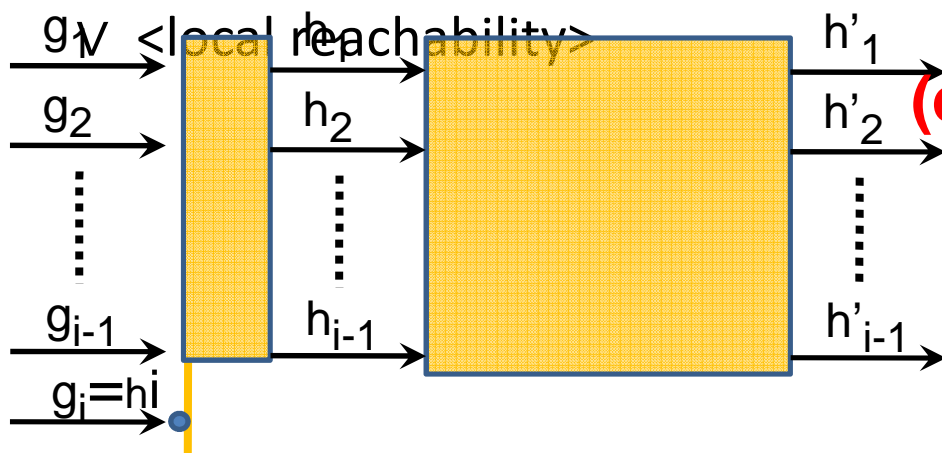
$\vee (i > 1 \wedge g_i = h_i \wedge \text{ThrdLI}(i - 1, \lambda, G, H)$

**(ensures lazy exploration for
the first round)**

$\wedge \exists H'. (\text{RightLI}(i - 1, H, H')$

$\wedge ((\text{SharedInit}(g_1) \wedge \text{Wrap}(G, H'))$

$\vee \text{WantRightLI}(i, G, H'))))$



**(starts contexts
(ensures lazy exploration for
explores each context in
the first round)**

Correctness

- Observe:
 - ThrdLI captures the actual exploration of program states
 - RightLI and WantRightLI: “service relations” for ensuring laziness and guide context-switching
- Context-bounded reachability is answered by checking

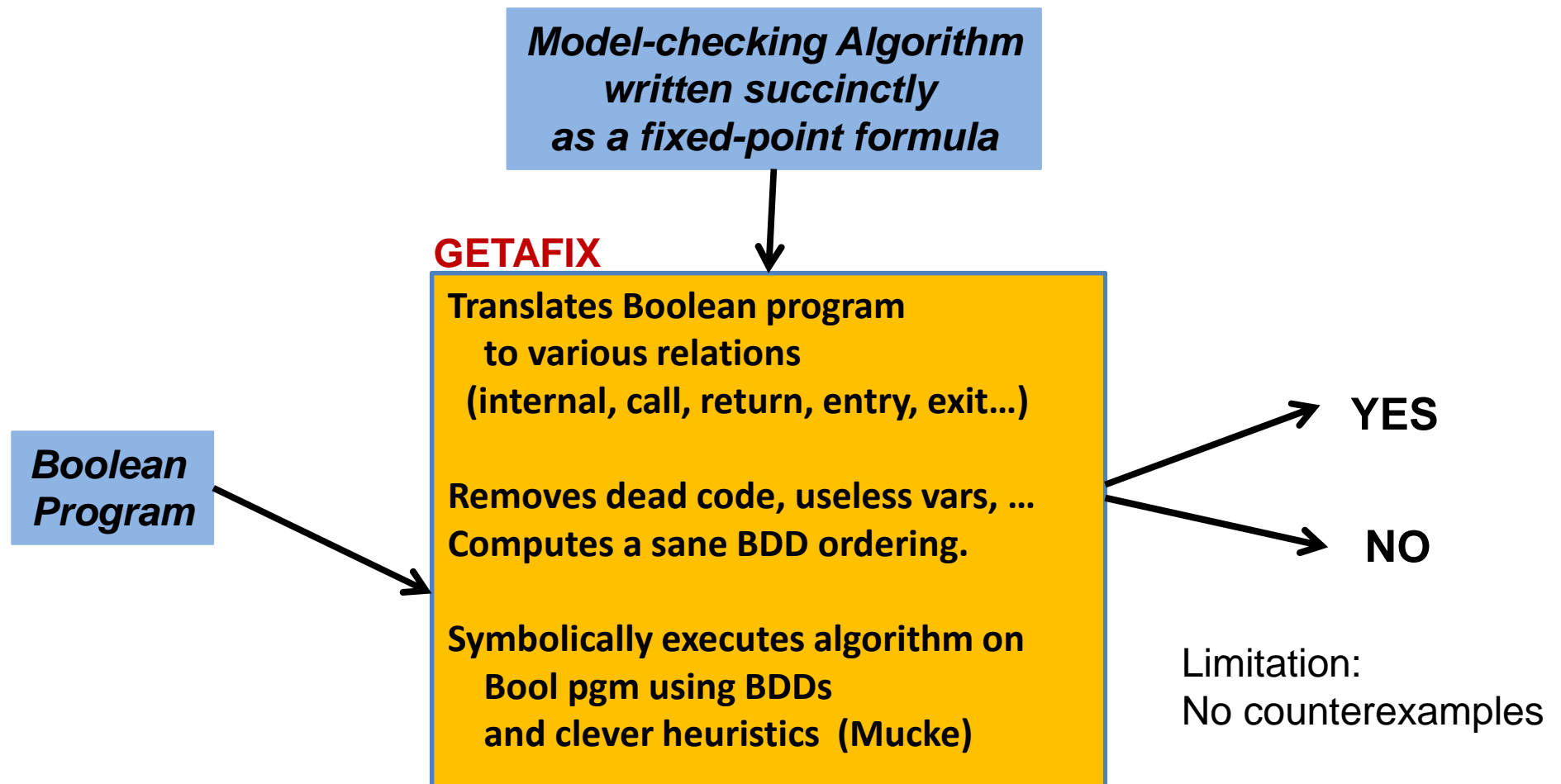
$\exists i, \lambda, G, H. (1 \leq i \leq k)$

$\wedge \text{ThrdLI}(i, \lambda, G, H) \wedge \text{Target}(\lambda)$

Concluding.....

Getafix

A framework for writing model-checking algorithms using fixed-point formulae.



Summary based approach

- BEBOP computes procedure summaries for Boolean programs [Ball-Rajamani, SPIN'00]
- First version of MOPED, an automaton accepting reachable stack configurations is constructed [Esparza-Schoown, CAV'01]
 - incrementally adds summary edges to the automaton until saturation
- Summaries for recursive state machines can be computed by DATALOG rules [Alur et al., TOPLAS'05]

Bounded context-switching

- [Quadeer,Wu, PLDI'04]: introduce bounded-context switch reachability (only 2 context-switches addressed)
- Algorithm [Qadeer-Rehof, TACAS'05] is complex and was implemented in [Suwimonteerabuth-Esparza-Schwoon,SPIN'08]
- [Suwimonteerabuth-Esparza-Schwoon,SPIN'08] implement algorithm of [Qadeer-Rehof, TACAS'05] and a new (simpler) version
- Algorithms presented in this lecture:
 - concurrent programs [La Torre-Madhusudan-Parlato, PLDI'09]
 - parameterized programs [La Torre-Madhusudan-Parlato, CAV'10]

Bounded context-switching (concurrent-to-sequential)

- Only the local state of one component program is kept at each time
 - multiple copies of shared variables
- [Lal-Reps, CAV'08]: translation to sequential reachability which yields eager state exploration
- [Lahiri-Qadeer-Rakamaric, CAV09]: use this translation for deductive verification of C programs
- [La Torre-Madhusudan-Parlato, CAV'09]: translation achieving lazy state exploration
 - preserves invariants of the concurrent program
- We also have eager and lazy translations for parameterized programs [La Torre-Madhusudan-Parlato, 2010 unpublished]